NEW APPROACH TO SOLVE FUZZY TRANSPORTATION PROBLEM WITH LR FLAT FUZZY NUMBERS

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Abstract :

The transportation problem is an important network structured linear programming problem (LPP) that arises in several contexts and has deservedly received considerable attention in the literature. The central concept in this problem is to determine the minimum total transportation cost of a commodity for satisfying the demand at destination using the available supply at the origins. The transportation problem can be applied to wide variety of situation, such as scheduling, production, investment, deciding plant location and inventory control. This paper presents a method to solve fuzzy transportation problem with the help of multi objective linear programming problem when the cost, supply and demand are LR Flat fuzzy in nature. Method is illustrated by the numerical examples and also compared with the existing methods.

Keywords: Fuzzy Transportation problem, LR flat fuzzy numbers, Fuzzy linear programming formulation, multi objective Linear Programming .

1.Introduction

The basic transportation problem was originally developed by Hitchcock [7]. In conventional transportation problem, it is assumed that decision maker is sure about the precise values of transportation cost, availability and demand of the product. In real world applications, all these parameters may not be known precisely due to uncontrollable factors. Fuzzy number introduced by Zadeh[14] may represent this data. So, fuzzy decision making method is needed here.

Zimmermann [15] showed that solutions obtained by fuzzy linear programming are always efficient. Subsequently, Zimmermann's fuzzy linear programming has developed into several fuzzy optimization methods for solving the transportation problems. Chanas et al [2] presented a fuzzy linear programming model for solving transportation problems with crisp cost coefficients and fuzzy supply and demand values. Chanas et al [3] formulated the fuzzy transportation problems (FTP) in three different situations and proposed method for solving the formulated fuzzy transportation problems. Chanas and Kuchta [4] proposed the concept of the optimal solution for the transportation problem with fuzzy coefficients expressed as fuzzy numbers, and developed an algorithm for obtaining the optimal solution. Chanas and Kuchta [5] proposed a new method for solving fuzzy integer transportation problem by representing the availability and demand parameters as L-R type fuzzy numbers.

A method involving a novel representation of trapezoidal fuzzy numbers was proposed by Kumar and Kaur [8] for finding the optimal solution of fuzzy transportation problems on the basis of fuzzy linear programming problems. In addition, a method based on the ranking function was proposed for solving the FTP assuming that transportation cost, supply and demand of the commodity are represented by generalized trapezoidal fuzzy numbers [9]. Moreover, a method was proposed for obtaining the exact fuzzy optimal solution of an unbalanced FTP by representing all the parameters as LR flat fuzzy numbers [10, 11]. Ali Ebrahimnejad [1] the proposed the method to solve Fuzzy Transportation Problem with LR Flat Fuzzy Numbers.

This paper is organised as follows : In section 2, some basic definitions are given. In section 3, linear programming formulation of fuzzy transportation problem is presented. In section 4, method to solve fuzzy transportation problem with LR flat fuzzy numbers using multi-objective linear programming is proposed. In section 5, numerical examples are provided to illustrate the proposed method and the solutions are compared with the existing methods.

2. Preliminaries

Here, we give some necessary definitions and results of fuzzy set theory given by [6]

Definition 1: An fuzzy number $\tilde{A}=(m,n,\alpha,\beta)_{LR}$ is said to be LR flat fuzzy number if

$$\mu_{\tilde{A}}(x)=\begin{cases} L\left(\frac{m-x}{\alpha}\right), & for \quad x\le m,\alpha>0, \\ R\left(\frac{x-n}{\beta}\right), & for \quad x\ge n,\beta>0 \\ 1, & otherwise \end{cases}$$

If m=n then $\tilde{A}=(m,n,\alpha,\beta)_{LR}$ will be converted into $\tilde{A}=(m,\alpha,\beta)_{LR}$ and is said to be LR fuzzy number. L and R are called reference functions, which are continuous, non-increasing functions that defines the left and right shapes of $\mu_{\tilde{A}}(x)$ respectively and $L(0)=R(0)=1$.

Definition 2: Let $\tilde{A}=(m,n,\alpha,\beta)_{LR}$ be a LR flat fuzzy number and λ be a real number in the interval [0,1] . Then the crisp set $A_{\lambda}=\left\{x\in X:\mu_{\tilde{A}}(x)\ge\lambda\right\}=\left[m-\alpha L^{-1}(\lambda),n+\beta R^{-1}(\lambda)\right]$ is said to be λ - cut of $\tilde{A}$.

Definition 3: An LR flat fuzzy number $\tilde{A}=(m,n,\alpha,\beta)_{LR}$ is said to be zero LR flat fuzzy number if and only if $m=0,n=0,\alpha=0,\beta=0.$

Definition 4: Two LR flat fuzzy numbers $\tilde{A}_1=(m_1,n_1,\alpha_1,\beta_1)_{LR}$ and $\tilde{A}_2=(m_2,n_2,\alpha_2,\beta_2)_{LR}$ are said to be equal. i.e., $\tilde{A}_1=\tilde{A}_2$ if and only if $m_1=m_2,n_1=n_2,\alpha_1=\alpha_2,\beta_1=\beta_2.$

Definition 5: An LR flat fuzzy number $\tilde{A}=(m,n,\alpha,\beta)_{LR}$ is said to be non- negative LR flat fuzzy number if $m-\alpha\ge 0.$

Definition 6: Let $\tilde{A}_1=(m_1,n_1,\alpha_1,\beta_1)_{LR}$ and $\tilde{A}_2=(m_2,n_2,\alpha_2,\beta_2)_{LR}$ be two LR flat fuzzy numbers. Then (i) $\tilde{A}_1\oplus\tilde{A}_2=(m_1+m_2,n_1+n_2,\alpha_1+\alpha_2,\beta_1+\beta_2)_{LR}$.

(ii) Let $\tilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ and $\tilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ be two non- negative LR flat fuzzy numbers. Then

$\tilde{A}_1 \otimes \tilde{A}_2 = (m_1 \cdot m_2, n_1 n_2, (m_1\alpha_2 + m_2\alpha_1), (n_1\beta_2 + n_2\beta_1))_{LR}.$

3. Linear Programming Formulation of fuzzy Transportation Problems

Fuzzy transportation problems can be converted into the following fuzzy linear programming Problem [10]:

Minimize $\sum_{i=1}^{p}\sum_{j=1}^{q} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$

Subject to $\sum_{j=1}^{q} \tilde{x}_{ij} = \tilde{a}_i, \qquad i = 1, 2, ..., p,$

$\sum_{i=1}^{p} \tilde{x}_{ij} = \tilde{b}_{j,} \qquad j = 1, 2, ..., q,$

x_{ij} is a non-negative LR flat fuzzy number,

Where, p = total number of sources; q = total number of destinations;

$\tilde{a}_i$ = the fuzzy availability of the product at i^{th} source;

$\tilde{b}_j$ = the fuzzy demand of the product at j^{th} destination;

$\tilde{C}_{ij}$ = the fuzzy transportation cost for one unit quantity of the product from i^{th} source to j^{th} destination; $\tilde{x}_{ij}$ = the fuzzy quantity of the product that should be transported from i^{th} source to j^{th} destination (or fuzzy decision variables) to minimize the total fuzzy transportation cost;

$\sum_{i=1}^{p} \tilde{a}_i$ = total fuzzy availability of the product; $\sum_{j=1}^{q} \tilde{b}_j$ = total fuzzy demand of the product;

$\sum_{i=1}^{p}\sum_{j=1}^{q} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$ = total fuzzy transportation cost.

4. Method to Solve Fuzzy Transportation Problem with LR Flat Fuzzy Numbers Using Multi-Objective Linear Programming

The steps of the method are as follows:

Step 1: Find the total fuzzy availability $\sum_{i=1}^{p} \tilde{a}_i$ and the total fuzzy demand $\sum_{j=1}^{q} \tilde{b}_j$. Let $\sum_{i=1}^{p} \tilde{a}_i = (m, n, \alpha, \beta)_{LR}$ and $\sum_{j=1}^{q} \tilde{b}_j = (m', n', \alpha', \beta')_{LR}$. Examine that the problem is balanced or not, i.e., $\sum_{i=1}^{p} \tilde{a}_i = \sum_{j=1}^{q} \tilde{b}_j$ or $\sum_{i=1}^{p} \tilde{a}_i \neq \sum_{j=1}^{q} \tilde{b}_j$

Case (i) If the problem is balanced, i.e., $\sum_{i=1}^{p} \tilde{a}_i = \sum_{j=1}^{q} \tilde{b}_j$, then go to step 2.

Case (ii) If $\sum_{i=1}^{p} \tilde{a}_i \neq \sum_{j=1}^{q} \tilde{b}_j$ then convert the unbalanced problem in to balanced problem as follows:

Case (a) If $m - \alpha \leq m' - \alpha', \alpha \leq \alpha', n - m \leq n' - m',$ and $\beta \leq \beta'$ then introduce a dummy source with fuzzy availability $(m' - m, n' - n, \alpha' - \alpha, \beta' - \beta)_{LR}$. Assume the fuzzy transportation cost

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for one unit quantity of the product from the introduced dummy source to all destinations as zero LR flat fuzzy number, then go to step 2.

Case (b) If $m-\alpha \le m'-\alpha', \alpha \le \alpha', n-m \le n'-m',$ and $\beta \le \beta'$ then introduce a dummy destination with fuzzy demand $(m'-m, n'-n, \alpha'-\alpha, \beta'-\beta)_{LR}$. Assume the fuzzy transportation cost for one unit quantity of the product from all sources to the introduced dummy destination as zero LR flat fuzzy number, then go to step 2.

Case (c) If neither case (a) nor case (b) is satisfied then introduce a dummy source with fuzzy availability(maximum$\{0,(m'-\alpha')-(m-\alpha)\}$ + maximum$\{0,(\alpha'-\alpha)\}$,
maximum$\{0,(m'-\alpha')-(m-\alpha)\}$ + maximum$\{0,(\alpha'-\alpha)\}$ + maximum$\{0,(n'-m')-(n-m)\}$,
maximum$\{0,(\alpha'-\alpha)\}$, maximum$\{0,(\beta'-\beta)\})_{LR}$
and dummy destination with fuzzy demand (maximum$\{0,(m-\alpha)-(m'-\alpha')\}$ + maximum $\{0,(\alpha-\alpha')\}$, maximum$\{0,(m-\alpha)-(m'-\alpha')\}$ + maximum$\{0,(\alpha-\alpha')\}$ +maximum $\{0,(n-m)-(n'-m'\}$, maximum$\{0,(\alpha-\alpha')\}$, maximum$\{0,(\beta-\beta')\})_{LR}$.

Assume the fuzzy transportation cost for one unit quantity of the product from the introduced dummy source to all destinations and from all sources to the introduced dummy destination as zero LR flat fuzzy number, then go to step 2.

Step 2: Formulate the balanced fuzzy transportation problem, obtained from step 1, into the following fuzzy linear programming problem:

$$\text{Minimize } \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij} \quad (1)$$

$$\text{Subject to } \sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_{i,} \quad i=1,2,3,\dots,m;\ m=p \text{ or } p+1, \quad (2)$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_{j,} \qquad j=1,2,3,\dots,n;\ n=q \text{ or } q+1,$$

$\tilde{x}_{ij}$ is a non-negative LR flat fuzzy number,

where m= total number of sources, n= total number of destinations, $\tilde{c}_{ij} = (m'_{ij}, n'_{ij}, \alpha'_{ij}, \beta'_{ij})_{,l}$
$\tilde{a}_i = (m_i, n_i, \alpha_i, \beta_i)_{LR,}$ $\tilde{b}_{ij} = (m'_j, n'_j, \alpha'_j, \beta'_j)_{LR,}$ $\tilde{x}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR,}$

Step 3: Convert the fuzzy linear programming problem with fuzzy coefficients, obtained from step 2, into the following fuzzy linear programming problem, using Arithmetic operation in definitions 4 and 6.

$$\text{Minimize } \left(\sum_{i=1}^{m}\sum_{j=1}^{n} (m'_{ij}, n'_{ij}, \alpha'_{ij}, \beta'_{ij}) \otimes (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})\right)$$

$$\text{Subject to} \left(\sum_{j=1}^{n} m_{ij}, \sum_{j=1}^{n} n_{ij}, \sum_{j=1}^{n} \alpha_{ij}, \sum_{j=1}^{n} \beta_{ij},\right)_{LR} = (m_i, n_i, \alpha_i, \beta_i)_{LR},\ i=1,2,3,\dots,m,$$

$$\left(\sum_{i=1}^{m} m_{ij}, \sum_{i=1}^{m} n_{ij}, \sum_{i=1}^{m} \alpha_{ij}, \sum_{i=1}^{m} \beta_{ij},\right)_{LR} = (m'_j, n'_j, \alpha'_j, \beta'_j)_{LR,}\ j=1,2,3,\dots,n.$$

$(m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}$ is a non-negative LR flat fuzzy number.

$$\text{Minimize} \sum_{i=1}^{m}\sum_{j=1}^{n} (m'_{ij}m_{ij}, n'_{ij}n_{ij}, (m'_{ij}\alpha_{ij} + m_{ij}\alpha'_{ij}), (n'_{ij}\beta_{ij} + n_{ij}\beta'_{ij}))$$

Subject to

$$\sum_{j=1}^{n} m_{ij} = m_{i,} \quad i = 1,2,3,...,m, \quad \sum_{j=1}^{n} n_{ij} = n_{i,} \quad i = 1,2,3,...,m,$$
$$\sum_{j=1}^{n} \alpha_{ij} = \alpha_{i,} \quad i = 1,2,3,...,m, \quad \sum_{j=1}^{n} \beta_{ij} = \beta_{i,} \quad i = 1,2,3,...,m,$$
$$\sum_{i=1}^{m} m_{ij} = m'_{j,} \quad j = 1,2,3,...,n., \quad \sum_{i=1}^{m} n_{ij} = n'_{j,} \quad j = 1,2,3,...,n.$$
$$\sum_{i=1}^{m} \alpha_{ij} = \alpha'_{j,} \quad j = 1,2,3,...,n., \quad \sum_{i=1}^{m} \beta_{ij} = \beta'_{j,} \quad j = 1,2,3,...,n. \tag{3}$$
$$m_{ij} - \alpha_{ij}, n_{ij} - m_{ij}, m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij} \geq 0 \qquad \forall i, j.$$

Step 4: Convert the fuzzy linear programming problem with fuzzy coefficients in step 3 into the following multi-objective linear programming problem [13] :

$$Minimize\left(f_1(x), f_2(x),..., f_k(x)\right)$$

Subject to (3)

where $f_i : R^n \rightarrow R^i$, where R is the set of all real numbers and R^n is the n-dimensional Euclidean space.

By considering the weighting factor, the MOLPP is defined as

$$Minimize_{x \in X}\ w = \sum_{m=1}^{k} w_m f_m(x) \tag{4}$$

Subject to (3)

Step 5: Solve the MOLPP linear programming problem, obtained from step 4,by giving weights to find the optimal solution $m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij}$.

Step 6: Find the fuzzy optimal solution $\tilde{x}_{ij}$ by putting the values of $m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij}$ in $\tilde{x}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR.}$

Step 7: Find the minimum total fuzzy transportation by putting the values of $\tilde{x}_{ij}$ in

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij} .$$

5. Numerical Examples

Example 1: [**11**] Data are collected from a trader that supplies a product (TMT), which is made from raw materials INGOT and BILLET, to different centers after obtaining the product from different plants. The trader supplies the product from three plants, Fortune Metals (MandiGobindgarh), KamdhenuSaria (Bhiwadi), and Goel Group (Raipur) to four different centers Ludhiana, Delhi, Himachal Pradesh, and LehLadakh. On the basis of the perception of the trader the approximate transportation cost per ton (in thousands of rupees), the approximate availability of the product (in tons) at different plants, and the approximate demand for the product (in tons) at different centers are represented by LR flat fuzzy numbers, as shown Table 1. Here L(x) =R(x) =max {0, 1-x}.

The trader wishes to determine the approximate quantity of the product that should be transported from each plant to each center so that the total approximate transportation cost in minimized. This problem can be formulated as the following

Source	**Destination**				**Supply**
	D_1	D_2	D_3	D_4	
S_1	$(20,21,1,1)_{LR}$	$(62,63,3,2)_{LR}$	$(95,97,5,2)_{LR}$	$(160,165,10,5)_{LR}$	$(3555,3580,55,420)_{LR}$
S_2	$(99,103,2,2)_{LR}$	$(17,19,2,2)_{LR}$	$(112,115,2,4)_{LR}$	$(210,220,20,20)_{LR}$	$(3175,3190,50,10)_{LR}$
S_3	$(262,264,2,6)_{LR}$	$(247,249,7,6)_{LR}$	$(274,279,2,11)_{LR}$	$(326,332,6,8)_{LR}$	$(2995,3275,520,125)_{LR}$
Demand	$(2500,2700,450,350)_{LR}$	$(3050,3100,50,100)_{LR}$	$(2150,2190,50,60)_{LR}$	$(2025,2055,75,45)_{LR}$	

Step 1: The problem is balanced, i.e., $\sum_{i=1}^{p}\tilde{a}_i = \sum_{j=1}^{q}\tilde{b}_j$, then go to step 2

Step 2: Let $\tilde{x}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})$ be the fuzzy quantity of the product that should be transported from the i^{th} source to the j^{th} destination so that the total fuzzy transportation cost is minimum. The balanced fuzzy transportation problem may be formulated into the following fuzzy linear programming problem:

$$\begin{aligned}\text{Minimize}(&(20,21,1,1)_{LR}\otimes\tilde{x}_{11}\oplus(62,63,3,2)_{LR}\otimes\tilde{x}_{12}\oplus(95,97,5,2)_{LR}\otimes\tilde{x}_{13}\oplus\\ &(160,165,10,5)_{LR}\otimes\tilde{x}_{14}\oplus(99,103,2,2)_{LR}\otimes\tilde{x}_{21}\oplus(17,19,2,2)_{LR}\otimes\tilde{x}_{22}\oplus\\ &(112,115,2,4)_{LR}\otimes\tilde{x}_{23}\oplus(210,220,20,20)_{LR}\otimes\tilde{x}_{24}\oplus(262,264,2,6)_{LR}\otimes\tilde{x}_{31}\oplus\\ &(247,249,7,6)_{LR}\otimes\tilde{x}_{32}\oplus(274,279,2,11)_{LR}\otimes\tilde{x}_{33}\oplus(326,332,6,8)_{LR}\otimes\tilde{x}_{34})\end{aligned}$$

Subject to

$$\begin{aligned}&\tilde{x}_{11}\oplus\tilde{x}_{12}\oplus\tilde{x}_{13}\oplus\tilde{x}_{14} = (3555,3580,55,420)_{LR}\\ &\tilde{x}_{21}\oplus\tilde{x}_{22}\oplus\tilde{x}_{23}\oplus\tilde{x}_{24} = (3175,3190,50,10)_{LR}\\ &\tilde{x}_{31}\oplus\tilde{x}_{32}\oplus\tilde{x}_{33}\oplus\tilde{x}_{34} = (2995,3275,520,125)_{LR}\\ &\tilde{x}_{11}\oplus\tilde{x}_{21}\oplus\tilde{x}_{31} = (2500,2700,450,350)_{LR}\\ &\tilde{x}_{12}\oplus\tilde{x}_{22}\oplus\tilde{x}_{32} = (3050,3100,50,100)_{LR}\\ &\tilde{x}_{13}\oplus\tilde{x}_{23}\oplus\tilde{x}_{33} = (2150,2190,50,60)_{LR}\\ &\tilde{x}_{14}\oplus\tilde{x}_{24}\oplus\tilde{x}_{34} = (2025,2055,75,45)_{LR}\end{aligned} \quad (5)$$

$\tilde{x}_{11,}\tilde{x}_{12,}\tilde{x}_{13,}\tilde{x}_{14,}\tilde{x}_{21,}\tilde{x}_{22,}\tilde{x}_{23,}\tilde{x}_{24,}\tilde{x}_{31,}\tilde{x}_{32,}\tilde{x}_{33,}\tilde{x}_{34,}$ arenon-negative LR flat fuzzy numbers.

$$\begin{aligned}&\text{(i.e,)Minimize}\left(20,21,1,1\right)\otimes(m_{11},n_{11},\alpha_{11},\beta_{11})\oplus\left(62,63,3,2\right)\otimes(m_{12},n_{12},\alpha_{12},\beta_{12})\oplus\\ &\left(95,97,5,2\right)\otimes(m_{13},n_{13},\alpha_{13},\beta_{13})\oplus\left(160,165,10,5\right)\otimes(m_{14},n_{14},\alpha_{14},\beta_{14})\\ &\oplus\left(99,103,2,2\right)\otimes(m_{21},n_{21},\alpha_{21},\beta_{21})\oplus\left(17,19,2,2\right)\otimes(m_{22},n_{22},\alpha_{22},\beta_{22})\\ &\oplus\left(112,115,2,4\right)\otimes(m_{23},n_{23},\alpha_{23},\beta_{23})\oplus\left(210,220,20,20\right)\otimes(m_{24},n_{24},\alpha_{24},\beta_{24})\end{aligned}$$

$$\oplus\left(262,264,2,6\right)\otimes(m_{31},n_{31},\alpha_{31},\beta_{31})\oplus\left(247,249,7,6\right)\otimes(m_{32},n_{32},\alpha_{32},\beta_{32})\oplus$$
$$\oplus\left(274,279,2,11\right)\otimes(m_{33},n_{33},\alpha_{33},\beta_{33})\oplus\left(326,332,6,8\right)\otimes(m_{34},n_{34},\alpha_{34},\beta_{34})$$

Subject to (5)

Step 3: Using Arithmetic Operations, the formulated fuzzy linear programming problem can be written in the following model**:**

Minimize

$$((20m_{11}\oplus 62m_{12}\oplus 95m_{13}\oplus 160m_{14}\oplus 99m_{21}\oplus 17m_{22}\oplus 112m_{23}\oplus 210m_{24}$$
$$\oplus 262m_{31}\oplus 247m_{32}\oplus 274m_{33}\oplus 326m_{34}),\ \ (21n_{11}\oplus 63n_{12}\oplus 97n_{13}\oplus 165n_{14}\oplus$$
$$103n_{21}\oplus 19n_{22}\oplus 115n_{23}\oplus 220n_{24}\oplus 264n_{31}\oplus 249n_{32}\oplus 279n_{33}\oplus 332n_{34}),$$
$$(m_{11}\oplus 3m_{12}\oplus 5m_{13}\oplus 10m_{14}\oplus 2m_{21}\oplus 2m_{22}\oplus 2m_{23}\oplus 20m_{24}$$
$$\oplus 2m_{31}\oplus 7m_{32}\oplus 2m_{33}\oplus 6m_{34}\oplus 20\alpha_{11}\oplus 62\alpha_{12}\oplus 95\alpha_{13}\oplus 160\alpha_{14}\oplus$$
$$99\alpha_{21}\oplus 17\alpha_{22}\oplus 112\alpha_{23}\oplus 210\alpha_{24}\oplus 262\alpha_{31}\oplus 247\alpha_{32}\oplus 274\alpha_{33}\oplus 326\alpha_{34}),$$
$$(n_{11}\oplus 2n_{12}\oplus 2n_{13}\oplus 5n_{14}\oplus 2n_{21}\oplus 2n_{22}\oplus 4n_{23}\oplus 20n_{24}$$
$$\oplus 6n_{31}\oplus 6n_{32}\oplus 11n_{33}\oplus 8n_{34}\oplus 21\beta_{11}\oplus 63\beta_{12}\oplus 97\beta_{13}\oplus 165\beta_{14}\oplus$$
$$103\beta_{21}\oplus 19\beta_{22}\oplus 115\beta_{23}\oplus 220\beta_{24}\oplus 264\beta_{31}\oplus 249\beta_{32}\oplus 279\beta_{33}\oplus 332\beta_{34}))$$
$$\oplus(326m_{34},332n_{34},6m_{34}+326\alpha_{34},8n_{34}+332\beta_{34})$$

Subject to

$$m_{11}+m_{12}+m_{13}+m_{14}=3555, n_{11}+n_{12}+n_{13}+n_{14}=3580,\ \alpha_{11}+\alpha_{12}+\alpha_{13}+\alpha_{14}=55,$$
$$\beta_{11}+\beta_{12}+\beta_{13}+\beta_{14}=420,\ m_{21}+m_{22}+m_{23}+m_{24}=3175, n_{21}+n_{22}+n_{23}+n_{24}=3190,$$
$$\alpha_{21}+\alpha_{22}+\alpha_{23}+\alpha_{24}=50, \beta_{21}+\beta_{22}+\beta_{23}+\beta_{24}=10, m_{31}+m_{32}+m_{33}+m_{34}=2995$$
$$m_{11}+m_{21}+m_{31}=2500, n_{11}+n_{21}+n_{31}=2700, \alpha_{11}+\alpha_{21}+\alpha_{31}=450,$$
$$\beta_{11}+\beta_{21}+\beta_{31}=350, m_{12}+m_{22}+m_{32}=3050, n_{12}+n_{22}+n_{32}=3100, \qquad (6)$$
$$\alpha_{12}+\alpha_{22}+\alpha_{32}=50, \beta_{12}+\beta_{22}+\beta_{32}=100, m_{13}+m_{23}+m_{33}=2150,$$
$$n_{13}+n_{23}+n_{33}=2190, \alpha_{13}+\alpha_{23}+\alpha_{33}=50, \beta_{13}+\beta_{23}+\beta_{33}=60, m_{14}+m_{24}+m_{34}=2025,$$
$$n_{14}+n_{24}+n_{34}=2055,\ \alpha_{14}+\alpha_{24}+\alpha_{34}=75,\ \beta_{14}+\beta_{24}+\beta_{34}=45.$$
$$n_{11}-m_{11}\geq 0,\quad m_{11}-\alpha_{11}\geq 0,\quad n_{12}-m_{12}\geq 0,\quad m_{12}-\alpha_{12}\geq 0,$$
$$n_{13}-m_{13}\geq 0,\quad m_{13}-\alpha_{13}\geq 0,\quad n_{14}-m_{14}\geq 0,\quad m_{14}-\alpha_{14}\geq 0,$$
$$n_{21}-m_{21}\geq 0,\quad m_{21}-\alpha_{21}\geq 0,\quad n_{22}-m_{22}\geq 0,\quad m_{22}-\alpha_{22}\geq 0,$$
$$n_{23}-m_{23}\geq 0,\quad m_{23}-\alpha_{23}\geq 0,\quad n_{24}-m_{24}\geq 0,\quad m_{24}-\alpha_{24}\geq 0,$$
$$n_{31}-m_{31}\geq 0,\quad m_{31}-\alpha_{31}\geq 0,\quad n_{32}-m_{32}\geq 0,\quad m_{32}-\alpha_{32}\geq 0,$$
$$n_{33}-m_{33}\geq 0,\quad m_{33}-\alpha_{33}\geq 0,\quad n_{34}-m_{34}\geq 0,\quad m_{34}-\alpha_{34}\geq 0,$$
$$m_{11},n_{11},\alpha_{11},\beta_{11},m_{12},n_{12},\alpha_{12},\beta_{12},m_{13},n_{13},\alpha_{13},\beta_{13},,m_{14},n_{14},\alpha_{14},\beta_{14},$$
$$m_{21},n_{21},\alpha_{21},\beta_{21},m_{22},n_{22},\alpha_{22},\beta_{22},m_{23},n_{23},\alpha_{23},\beta_{23},,m_{24},n_{24},\alpha_{24},\beta_{24},$$
$$m_{31},n_{31},\alpha_{31},\beta_{31},m_{32},n_{32},\alpha_{32},\beta_{32},m_{33},n_{33},\alpha_{33},\beta_{33},,m_{34},n_{34},\alpha_{34},\beta_{34}\geq 0.$$

Using Steps 4 and 5, the above FLPP can be converted to MOLPP (giving $w_1 = w_2 = w_3 =$ w4=1)as follows

Minimize

W= $(21m_{11} \oplus 65m_{12} \oplus 100m_{13} \oplus 170m_{14} \oplus 101m_{21} \oplus 19m_{22} \oplus 114m_{23} \oplus 230m_{24}$
$\oplus 264m_{31} \oplus 254m_{32} \oplus 276m_{33} \oplus 332m_{34} \oplus 22n_{11} \oplus 65n_{12} \oplus 99n_{13} \oplus 170n_{14} \oplus$
$105n_{21} \oplus 21n_{22} \oplus 119n_{23} \oplus 240n_{24} \oplus 270n_{31} \oplus 255n_{32} \oplus 290n_{33} \oplus 340n_{34}$
$\oplus 20\alpha_{11} \oplus 62\alpha_{12} \oplus 95\alpha_{13} \oplus 160\alpha_{14} \oplus 99\alpha_{21} \oplus 17\alpha_{22} \oplus 112\alpha_{23} \oplus 210\alpha_{24}$
$\oplus 262\alpha_{31} \oplus 247\alpha_{32} \oplus 274\alpha_{33} \oplus 326\alpha_{34} \oplus 21\beta_{11} \oplus 63\beta_{12} \oplus 97\beta_{13} \oplus 165\beta_{14} \oplus$
$103\beta_{21} \oplus 19\beta_{22} \oplus 115\beta_{23} \oplus 220\beta_{24} \oplus 264\beta_{31} \oplus 249\beta_{32} \oplus 279\beta_{33} \oplus 332\beta_{34})$

Subject to (6)

Then the optimal solution of the above crisp linear programming problem using LINGO is obtained as follows

$m_{11} = 2105, n_{11} = 2130, \alpha_{11} = 55, \beta_{11} = 350,\ m_{12} = 0, n_{12} = 0, \alpha_{12}, 0, \beta_{12} = 70,$
$m_{13} = 1450, n_{13} = 1450, \alpha_{13} = 0, \beta_{13} = 0,\ m_{14} = 0, n_{14} = 0, \alpha_{14} = 0, \beta_{14} = 0,$
$m_{21} = 0, n_{21} = 0, \alpha_{21} = 0, \beta_{21} = 0,\ m_{22} = 3050, n_{22} = 3065, \alpha_{22} = 50, \beta_{22} = 10,$
$m_{23} = 125, n_{23} = 125, \alpha_{23} = 0, \beta_{23} = 0, m_{24} = 0, n_{24} = 0, \alpha_{24} = 0, \beta_{24} = 0,$
$m_{31} = 395, n_{31} = 570, \alpha_{31} = 395, \beta_{31} = 0, m_{32} = 0, n_{32} = 35, \alpha_{32} = 0, \beta_{32} = 20,$
$m_{33} = 575, n_{33} = 615, \alpha_{33} = 50, \beta_{33} = 60, m_{34} = 2025, n_{34} = 2055, \alpha_{34} = 75, \beta_{34} = 45.$

Putting the value of $m_{ij}, n_{ij}, \alpha_{ij}$ *and* β_{ij} in $\tilde{x}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}$, the fuzzy optimal solution is
$\tilde{x}_{11} = (2105, 2130, 55, 350)_{LR}$, $\tilde{x}_{12} = (0,0,0,0)_{LR}$, $\tilde{x}_{13} = (1450, 1450, 0, 0)_{LR}$, $\tilde{x}_{14} = (0,0,0,0)_{LR}$,
$\tilde{x}_{21} = (0,0,0,0)_{LR}$, $\tilde{x}_{22} = (3050, 3065, 50, 10)_{LR}$, $\tilde{x}_{23} = (125, 125, 0, 0)_{LR}$, $\tilde{x}_{24} = (0,0,0,0)_{LR}$,
$\tilde{x}_{31} = (395, 570, 395, 0)_{LR}$, $\tilde{x}_{32} = (0, 35, 0, 20)_{LR}$, $\tilde{x}_{33} = (575, 615, 50, 60)_{LR}$,
$\tilde{x}_{34} = (2025, 2055, 75, 45)_{LR}$. Then

$$\tilde{\sum_{i=1,2,3}} \tilde{\sum_{j=1,2,3,4}} \tilde{c}_{ij} \otimes \tilde{x}_{ij} = (1166890, 1271030, 173385, 82645)_{LR}$$

Example: 2 [10]

A company has two sources S_1 and S_2 and three destinations D_1, D_2 and D_3; the fuzzy transportation cost for unit quantity of the product from i^{th} source to j^{th} destination is $\tilde{c}_{ij}$, where

$$\left[\tilde{c}_{ij}\right]_{2X3} = \begin{bmatrix} (20,30,10,10)_{LR} & (60,70,10,20)_{LR} & (90,110,10,10)_{LR} \\ (70,80,10,10)_{LR} & (80,100,10,20)_{LR} & (30,50,10,10)_{LR} \end{bmatrix}$$

The fuzzy availability of the product at first and second sources are $(90,90,20,10)_{LR}$ and $(60,70,20,10)_{LR}$ and the fuzzy demand of the product at the first, second and third destinations is $(40,50,10,20)_{LR}$, $(30,40,10,10)_{LR}$ and $(50,50,10,30)_{LR}$, respectively, here, L(x)=R(x)=max{0,1-x}. The company want to determine the fuzzy quantity of the product that should be transported from each of the sources to each destination so that the total fuzzy transportation cost is minimum.

Step 1: Total fuzzy availability $= (150, 160, 40, 20)_{LR}$ and total fuzzy demand= $(120, 140, 30, 60)_{LR}$

Since total availability $\neq$ total fuzzy demand, it is an unbalanced fuzzy transportation problem.
Now as described in the method (using Case (c) of Step 1 of the method), the unbalanced fuzzy transportation problem can be converted into a balanced fuzzy transportation problem, by introducing dummy source S_3 with fuzzy availability $(0,10,0,40)_{LR}$ and a dummy destination D_4 with fuzzy demand $(30,30,10,0)_{LR}$, so that total fuzzy availability = total fuzzy demand, i.e. $(150,160,40,20)_{LR} \oplus (0,10,0,40)_{LR} = (120,140,30,60)_{LR} \oplus (30,30,10,0)_{LR}$.
Assuming the fuzzy transportation cost for one unit quantity of the product from dummy source S_3 to all destinations and from all sources to dummy destination D_4 as zero LR flat fuzzy number, i.e., $\tilde{c}_{14} = \tilde{c}_{24} = \tilde{c}_{31} = \tilde{c}_{32} = \tilde{c}_{33} = \tilde{c}_{34} = (0,0,0,0)_{LR}$.

Destination → **Source ↓**	D_1	D_2	D_3	D_4	
S_1	$(20,30,10,10)_{LR}$	$(60,70,10,20)_{LR}$	$(90,110,10,10)_{LR}$	$(0,0,0,0)_{LR}$	$(90,90,20,10)_{LR}$
S_2	$(70,80,10,10)_{LR}$	$(80,100,10,20)_{LR}$	$(30,50,10,10)_{LR}$	$(0,0,0,0)_{LR}$	$(60,70,20,10)_{LR}$
S_3	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,10,0,40)_{LR}$
	$(40,50,10,20)_{LR}$	$(30,40,10,10)_{LR}$	$(50,50,10,30)_{LR}$	$(30,30,10,0)_{LR}$	

Step 2: Let $\tilde{x}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})$ be the fuzzy quantity of the product that should be transported from the i^{th} source to the j^{th} destination so that the total fuzzy transportation cost is minimum. The obtained balanced fuzzy transportation problem may be formulated into the following fuzzy linear programming problem:

Minimize($(20,30,10,10)_{LR} \otimes \tilde{x}_{11} \oplus (60,70,10,20)_{LR} \otimes \tilde{x}_{12} \oplus (90,110,10,10)_{LR} \otimes \tilde{x}_{13} \oplus$
$(0,0,0,0)_{LR} \otimes \tilde{x}_{14} \oplus (70,80,10,10)_{LR} \otimes \tilde{x}_{21} \oplus (80,100,10,20)_{LR} \otimes \tilde{x}_{22} \oplus$
$(30,50,10,10)_{LR} \otimes \tilde{x}_{23} \oplus (0,0,0,0)_{LR} \otimes \tilde{x}_{24} \oplus (0,0,0,0)_{LR} \otimes \tilde{x}_{31} \oplus$
$(0,0,0,0)_{LR} \otimes \tilde{x}_{32} \oplus (0,0,0,0)_{LR} \otimes \tilde{x}_{33} \oplus (0,0,0,0)_{LR} \otimes \tilde{x}_{34}$)

Subject to

$$
\begin{aligned}
&\tilde{x}_{11} \oplus \tilde{x}_{12} \oplus \tilde{x}_{13} \oplus \tilde{x}_{14} = (90,90,20,10)_{LR} \\
&\tilde{x}_{21} \oplus \tilde{x}_{22} \oplus \tilde{x}_{23} \oplus \tilde{x}_{24} = (60,70,20,10)_{LR} \\
&\tilde{x}_{31} \oplus \tilde{x}_{32} \oplus \tilde{x}_{33} \oplus \tilde{x}_{34} = (0,10,0,40)_{LR} \\
&\tilde{x}_{11} \oplus \tilde{x}_{21} \oplus \tilde{x}_{31} = (40,50,10,20)_{LR} \qquad (7) \\
&\tilde{x}_{12} \oplus \tilde{x}_{22} \oplus \tilde{x}_{32} = (30,40,10,10)_{LR} \\
&\tilde{x}_{13} \oplus \tilde{x}_{23} \oplus \tilde{x}_{33} = (50,50,10,30)_{LR} \\
&\tilde{x}_{14} \oplus \tilde{x}_{24} \oplus \tilde{x}_{34} = (30,30,10,0)_{LR}
\end{aligned}
$$

$\tilde{x}_{11}, \tilde{x}_{12}, \tilde{x}_{13}, \tilde{x}_{14}, \tilde{x}_{21}, \tilde{x}_{22}, \tilde{x}_{23}, \tilde{x}_{24}, \tilde{x}_{31}, \tilde{x}_{32}, \tilde{x}_{33}, \tilde{x}_{34},$ arenon-negative LR flat fuzzy numbers.

Using step3 we get,

$$Minimize\,((20m_{11} \oplus 60m_{12} \oplus 90m_{13} \oplus 0m_{14} \oplus 70m_{21} \oplus 80m_{22} \oplus 30m_{23} \oplus 0m_{24}$$
$$\oplus 0m_{31} \oplus 0m_{32} \oplus 0m_{33} \oplus 0m_{34}),\ \ (30n_{11} \oplus 70n_{12} \oplus 110n_{13} \oplus 0n_{14} \oplus$$
$$80n_{21} \oplus 100n_{22} \oplus 50n_{23} \oplus 0n_{24} \oplus 0n_{31} \oplus 0n_{32} \oplus 0n_{33} \oplus 0n_{34}),$$
$$(10m_{11} \oplus 10m_{12} \oplus 10m_{13} \oplus 0m_{14} \oplus 10m_{21} \oplus 10m_{22} \oplus 10m_{23} \oplus 0m_{24}$$
$$\oplus 0m_{31} \oplus 0m_{32} \oplus 0m_{33} \oplus 0m_{34} \oplus 20\alpha_{11} \oplus 60\alpha_{12} \oplus 90\alpha_{13} \oplus 0\alpha_{14} \oplus$$
$$70\alpha_{21} \oplus 80\alpha_{22} \oplus 30\alpha_{23} \oplus 0\alpha_{24} \oplus 0\alpha_{31} \oplus 0\alpha_{32} \oplus 0\alpha_{33} \oplus 0\alpha_{34}),$$
$$(10n_{11} \oplus 20n_{12} \oplus 10n_{13} \oplus 0n_{14} \oplus 10n_{21} \oplus 20n_{22} \oplus 10n_{23} \oplus 0n_{24}$$
$$\oplus 0n_{31} \oplus 0n_{32} \oplus 0n_{33} \oplus 0n_{34} \oplus 30\beta_{11} \oplus 70\beta_{12} \oplus 110\beta_{13} \oplus 0\beta_{14} \oplus$$
$$80\beta_{21} \oplus 100\beta_{22} \oplus 50\beta_{23} \oplus 0\beta_{24} \oplus 0\beta_{31} \oplus 0\beta_{32} \oplus 0\beta_{33} \oplus 0\beta_{34}))$$

Subject to

$$m_{11}+m_{12}+m_{13}+m_{14}=90, \qquad n_{11}+n_{12}+n_{13}+n_{14}=90,$$
$$\alpha_{11}+\alpha_{12}+\alpha_{13}+\alpha_{14}=20, \qquad \beta_{11}+\beta_{12}+\beta_{13}+\beta_{14}=10,$$
$$m_{21}+m_{22}+m_{23}+m_{24}=60, \qquad n_{21}+n_{22}+n_{23}+n_{24}=70, \qquad (8)$$
$$\alpha_{21}+\alpha_{22}+\alpha_{23}+\alpha_{24}=20,, \qquad \beta_{21}+\beta_{22}+\beta_{23}+\beta_{24}=10,$$
$$m_{31}+m_{32}+m_{33}+m_{34}=0, n_{31}+n_{32}+n_{33}+n_{34}=10,\ \alpha_{31}+\alpha_{32}+\alpha_{33}+\alpha_{34}=0,$$
$$\beta_{31}+\beta_{32}+\beta_{33}+\beta_{34}=40, m_{11}+m_{21}+m_{31}=40, n_{11}+n_{21}+n_{31}=50,$$
$$\alpha_{11}+\alpha_{21}+\alpha_{31}=10,\ \beta_{11}+\beta_{21}+\beta_{31}=20,\ m_{12}+m_{22}+m_{32}=30,\ n_{12}+n_{22}+n_{32}=40,$$
$$\alpha_{12}+\alpha_{22}+\alpha_{32}=10, \beta_{12}+\beta_{22}+\beta_{32}=10, m_{13}+m_{23}+m_{33}=50, n_{13}+n_{23}+n_{33}=50,$$
$$\alpha_{13}+\alpha_{23}+\alpha_{33}=10, \beta_{13}+\beta_{23}+\beta_{33}=30, m_{14}+m_{24}+m_{34}=30, n_{14}+n_{24}+n_{34}=30,$$
$$\alpha_{14}+\alpha_{24}+\alpha_{34}=10, \beta_{14}+\beta_{24}+\beta_{34}=0. \qquad .$$
$$n_{11}-m_{11}\ge 0,\quad m_{11}-\alpha_{11}\ge 0,\quad n_{12}-m_{12}\ge 0,\quad m_{12}-\alpha_{12}\ge 0,$$
$$n_{13}-m_{13}\ge 0,\quad m_{13}-\alpha_{13}\ge 0,\quad n_{14}-m_{14}\ge 0,\quad m_{14}-\alpha_{14}\ge 0,$$
$$n_{21}-m_{21}\ge 0,\quad m_{21}-\alpha_{21}\ge 0,\quad n_{22}-m_{22}\ge 0,\quad m_{22}-\alpha_{22}\ge 0,$$
$$n_{23}-m_{23}\ge 0,\quad m_{23}-\alpha_{23}\ge 0,\quad n_{24}-m_{24}\ge 0,\quad m_{24}-\alpha_{24}\ge 0,$$
$$n_{31}-m_{31}\ge 0,\quad m_{31}-\alpha_{31}\ge 0,\quad n_{32}-m_{32}\ge 0,\quad m_{32}-\alpha_{32}\ge 0,$$
$$n_{33}-m_{33}\ge 0,\quad m_{33}-\alpha_{33}\ge 0,\quad n_{34}-m_{34}\ge 0,\quad m_{34}-\alpha_{34}\ge 0,$$
$$m_{11},n_{11},\alpha_{11},\beta_{11},m_{12},n_{12},\alpha_{12},\beta_{12},m_{13},n_{13},\alpha_{13},\beta_{13},,m_{14},n_{14},\alpha_{14},\beta_{14},$$
$$m_{21},n_{21},\alpha_{21},\beta_{21},m_{22},n_{22},\alpha_{22},\beta_{22},m_{23},n_{23},\alpha_{23},\beta_{23},,m_{24},n_{24},\alpha_{24},\beta_{24},$$
$$m_{31},n_{31},\alpha_{31},\beta_{31},m_{32},n_{32},\alpha_{32},\beta_{32},m_{33},n_{33},\alpha_{33},\beta_{33},,m_{34},n_{34},\alpha_{34},\beta_{34}\ge 0.$$

Using Steps 4 and 5, the above FLPP can be converted to MOLPP (giving $w_1 = w_2 = w_3 = w4=1$) as follows

$$Minimize\,(30m_{11} \oplus 70m_{12} \oplus 100m_{13} \oplus 0m_{14} \oplus 80m_{21} \oplus 90m_{22} \oplus 40m_{23} \oplus 0m_{24}$$
$$\oplus 0m_{31} \oplus 0m_{32} \oplus 0m_{33} \oplus 0m_{34} \oplus 40n_{11} \oplus 90n_{12} \oplus 120n_{13} \oplus 0n_{14} \oplus$$
$$90n_{21} \oplus 120n_{22} \oplus 60n_{23} \oplus 0n_{24} \oplus 0n_{31} \oplus 0n_{32} \oplus 0n_{33} \oplus 0n_{34} \oplus$$
$$20\alpha_{11} \oplus 60\alpha_{12} \oplus 90\alpha_{13} \oplus 0\alpha_{14} \oplus 70\alpha_{21} \oplus 80\alpha_{22} \oplus 30\alpha_{23} \oplus 0\alpha_{24} \oplus$$
$$0\alpha_{31} \oplus 0\alpha_{32} \oplus 0\alpha_{33} \oplus 0\alpha_{34} \oplus 30\beta_{11} \oplus 70\beta_{12} \oplus 110\beta_{13} \oplus 0\beta_{14} \oplus$$
$$80\beta_{21} \oplus 100\beta_{22} \oplus 50\beta_{23} \oplus 0\beta_{24} \oplus 0\beta_{31} \oplus 0\beta_{32} \oplus 0\beta_{33} \oplus 0\beta_{34})$$

Subject to (8)
Then the optimal solution of the above crisp linear programming problem using LINGO is obtained as follows

$m_{11}=40, n_{11}=40, \alpha_{11}=10, \beta_{11}=10,\ m_{12}=30, n_{12}=30, \alpha_{12}=10, \beta_{12}=0,$

$m_{13}=0, n_{13}=0, \alpha_{13}=0, \beta_{13}=0, \quad m_{14}=20, n_{14}=20, \alpha_{14}=0, \beta_{14}=0,$

$m_{21}=0, n_{21}=10, \alpha_{21}=0, \beta_{21}=0,\ m_{22}=0, n_{22}=0, \alpha_{22}=0, \beta_{22}=0,$

$m_{23}=50, n_{23}=50, \alpha_{23}=10, \beta_{23}=10,\ m_{24}=10, n_{24}=10, \alpha_{24}=10,$

$\beta_{24}=0,\ m_{31}=0, n_{31}=0, \alpha_{31}=0, \beta_{31}=10,\ m_{32}=0, n_{32}=10, \alpha_{32}=0,$

$\beta_{32}=10, m_{33}=0, n_{33}=0, \alpha_{33}=0, \beta_{33}=20, m_{34}=0, n_{34}=0, \alpha_{34}=0,$

$\beta_{34}=0.$

Putting the value of $m_{ij}, n_{ij}, \alpha_{ij}$ *and* β_{ij} in $\tilde{x}_{ij}=(m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}$, the fuzzy optimal solution is

$\tilde{x}_{11}=(40,40,10,10)_{LR},\ \tilde{x}_{12}=(30,30,10,0)_{LR},\ \tilde{x}_{13}=(0,0,0,0)_{LR},\ \tilde{x}_{14}=(20,20,0,0)_{LR},$

$\tilde{x}_{21}=(0,10,0,0)_{LR},\ \tilde{x}_{22}=(0,0,0,0)_{LR},\ \tilde{x}_{23}=(50,50,10,10)_{LR},\ \tilde{x}_{24}=(10,10,10,0)_{LR},$

$\tilde{x}_{31}=(0,0,0,10)_{LR},\ \tilde{x}_{32}=(0,10,0,10)_{LR},\ \tilde{x}_{33}=(0,0,0,20)_{LR},\ \tilde{x}_{34}=(0,0,0,0)_{LR}.$

And the minimum total fuzzy transportation cost is $\left(4100, 6600, 2000, 2600\right)_{LR}$.

Comparison Table

The following table gives the optimal solution of the problems discussed in the section 5 by the existing methods ([10], [1])

Methods	Example: 1	Example: 2
Kumar A, Kaur A[10](method based on linear programming formulation)	(1166890,1271030,173385,82645)LR	(4100,6600,2000,2600)*LR*
Kumar A, Kaur A[10](method based on classical transportation methods)	(1166890,1271030,173385,82645)LR	(4100,6600,2000,2600)*LR*
Ali Ebrahimnejad[1]	(999500,1166890,1271030,1359725)LR	(2100,4100,6600,9200)LR
Proposed Method	(1166890,1271030,173385,82645)LR	(4100,6600,2000,2600)*LR*

From the above table it is observed that the solution of proposed method is similar to the existing methods.

6. Conclusion :

Here, a method to solve Fuzzy Transportation problem with LR flat fuzzy numbers using multi-objective linear programming is proposed. Numerical examples are provided to illustrate the method and finally the solutions are compared with solutions of the existing methods and observed that the solution of proposed method is similar to the existing methods.

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