Pythagorean Vague Binary Soft Sets

Remya.P.B #1, Dr.Francina Shalini.A *2
#1 Ph D Research Scholar, *2 Assistant Professor
Department of Mathematics,
Nirmala College for Women, Affiliated to Bharathiar University,
Red Fields, Coimbatore-18, Tamil Nadu
1 krish3thulasi@gmail.com
2 francshalu@gmail.com

Abstract— **Pythagorean vague binary soft sets are developed in this paper. Various distance measures are mentioned with example and application. It's higher dimension stage q-rung orthopair vague binary soft sets are also discussed.**

***Keywords*— Pythagorean vague soft set, Pythagorean vague binary soft set, Operations on pythagorean vague binary soft sets, distance measure, q-rung orthopair vague binary soft set**

I. Introduction

Inadequacy of George Cantor's [4] Classical set theory made researchers to seek new tools to handle complex real life situations. Zadeh [4] succeeded in that and introduced fuzzy set theory in 1965, in which partial membership is allowed. Later in 1986, Attanassov [4] introduced classical intuitionistic fuzzy sets. IFS's helped to handle the uncertainty or hesitation region of fuzzy sets. He also introduced intuitionistic fuzzy sets (IFS's) of second type [3] in 1989 in which square sum of the membership grades is less than or equal to one. This sum takes greater than or equal to one in some real situations which demands some other tools to overcome this difficulty. Thus Pythagorean fuzzy sets [8, 9, 10, 14] arose which handled both these situations flexibly. Pythagorean aspects were firstly used in fuzzy set theory. Thus Pythagorean fuzzy sets are introduced in 2016 by Yager [10] to extend intuitionistic fuzzy sets. Later intuitionistic pythagorean fuzzy sets [10] are introduced. q-rung orthopair fuzzy set (q - ROFS with $q \geqslant 3$) is developed by yager [13] in 2018. Comparing with pythagorean fuzzy set it seems to be more stronger when dealing with uncertainty problems. In some situations membership degrees take values inside an interval and not a precise one. Interval valued pythagorean fuzzy set introduced by X.Peng [9] handled these situations more reliably. Vague sets were proposed by Gau and Buehrer [1, 2, 4] as an extension of fuzzy set theory. Pythagorean vague sets are introduced by Nirmala Irudayam [11] and Vinnarasi in 2018. Duojie et al., [3] introduced possibility pythagorean fuzzy soft set and its application in 2019. Pythagorean fuzzy number is introduced by Zhang and Xu [11]. They also introduced comparison laws for Pythagorean fuzzy numbers. Weakness of the existing correlation coefficients in intuitionistic fuzzy set theory made Harish Garg [5] to develop a novel correlation coefficient in pythagorean fuzzy sets in 2016. Later Harish Garg [6] introduced hesitant pythagorean fuzzy set by combining the concepts pythagorean and hesitant fuzzy set in 2018. Vague binary soft sets were introduced by Dr.Francina Shalini.A [4] and Remya.P.B. in 2018. They discussed some of it's properties. Pythagorean membership grade takes more space than intuitionistic membership grades. An intuitionistic membership grade is always Pythagorean but the converse need not be! Pythagorean vague membership grade is a point on a unit circle. This paper aims to develop Pythagorean nature of vague binary soft sets via Pythagorean vague sets and vague binary soft sets. Some distance measures are developed. Using that a decision making problem is also discussed. Besides it's higher stage, q-rung orthopair vague binary soft sets are also developed.

II. PRELIMINARIES

In this section, some preliminaries are given.

Definition 2.1. [4] (Vague binary soft set) Let $\{U_1, U_2\}$ be two initial universes which is common to a fixed set A⊆ E of parameters. Let V(U_1), V(U_2) denote the power set of vague sets on $\{U_1, U_2\}$ respectively. A pair ($\ddot{F}$, A) is said to be a vague binary soft set [VBSS]over $\{U_1, U_2\}$ where $\ddot{F}$ is a mapping given by $\ddot{F}$: A → V (U_1)×V (U_2) and ($\ddot{F}$, A) = $\{e_i \in$ A/(e_i, $\ddot{F}(e_i)$)}; where $\ddot{F}(e_i) = \left(\langle\frac{V_{\ddot{F}(e_i)}(x_j)}{x_j};\ \forall\, e_i \in \text{A}, \forall\, x_j \in U_1\rangle, \langle\frac{V_{\ddot{F}(e_i)}(y_k)}{y_k}; \forall\, e_i \in \text{A}, \forall\, y_k \in U_2\ \rangle\right)$;

$V_{\ddot{F}(e_i)}(x_j) = [t_{\ddot{F}(e_i)}(x_j), 1- f_{\ddot{F}(e_i)}(x_j)]$; $V_{\ddot{F}(e_i)}(y_k) = [t_{\ddot{F}(e_i)}(y_k), 1- f_{\ddot{F}(e_i)}(y_k)]$

Definition 2.2. [11] (Pythagorean vague set) Let X be a universe of discourse. A Pythagorean Vague set (PVS), A in X is given by A = {< x, $t_A(x_i)$, 1−$f_A(x_i)$ / x_i∈ X}, where $t_A(x_i)$: X → [0,1] denotes the truth value and 1−$f_A(x_i)$: X → [0,1] denotes the false value of the element x_i∈ X to the set A, respectively, with the condition that $0 \leqslant (t_A(x_i))^2 + (1-f_A(x_i))^2 \leqslant 1$

Definition 2.3.[7] (Pythagorean fuzzy soft set) Given an initial universe set U and a universe set of parameters E. A pair (F, A) is referred to as a Pythagorean fuzzy soft set (PFSS) on U if A ⊆ E and F: A → PF(U), where PF(U) is the family of all Pythagorean fuzzy subsets of U.

Definition 2.4. [12] (Szmidt and Kaeprzyk's Distances between Vague Sets)

d^{H}_{Sz} (A,B) = $\frac{1}{2}\sum_{i=1}^{n}[|\Delta_t(\text{i})| + |\Delta_f(\text{i})| + |\Delta_\Pi(\text{i})|]$

d^{nH}_{Sz} (A,B) = $\frac{1}{2n}\sum_{i=1}^{n}[|\Delta_t(\text{i})| + |\Delta_f(\text{i})| + |\Delta_\Pi(\text{i})|]$

d^{E}_{Sz} (A,B) = $\sqrt{\frac{1}{2}\sum_{i=1}^{n}[|\Delta_t(\text{i})|^2 + |\Delta_f(\text{i})|^2 + |\Delta_\Pi(\text{i})|^2]}$

d^{nE}_{Sz} (A,B) = $\sqrt{\frac{1}{2n}\sum_{i=1}^{n}[|\Delta_t(\text{i})|^2 + |\Delta_f(\text{i})|^2 + |\Delta_\Pi(\text{i})|^2]}$

where Δ_t(i)) = $t_A(x_i) - t_B(x_i)$, Δ_f(i) = $f_A(x_i) - f_B(x_i)$, Δ_Π(i)) = $\Pi_A(x_i) - \Pi_B(x_i)$

Definition 2.5. [11](Various distances for Pythagorean vague set)

Let A = {⟨x, $(t_A(x_i), 1 - f_A(x_i))$⟩ / x_i ={x_1, x_2, --------, x_n}∈ X} and

B = {⟨ x, ($t_B(x_i), 1 - f_B(x_i)$)⟩ / x_i={x_1, x_2, --------, x_n}∈ X} be Pythagorean vague sets in X.

1. Hamming distance H_{PVS}(A, B) = $\frac{1}{2}\sum_{i=1}^{n}(|t_A(x_i) - t_B(x_i)| + |(1 - f_A(x_i)) - (1 - f_B(x_i))| + |\Pi_A(x_i) - \Pi_B(x_i)|)$
2. Normalized hamming distance N_{PVS}(A,B)=$\frac{1}{2n}\sum_{i=1}^{n}(|t_A(x_i) - t_B(x_i)| + |(1 - f_A(x_i)) - (1 - f_B(x_i))| + |\Pi_A(x_i) - \Pi_B(x_i)|)$
3. Euclidean distance E_{PVS}(A, B)=$\sqrt{\frac{1}{2}\sum_{i=1}^{n}(|t_A(x_i) - t_B(x_i)|^2 + |(1 - f_A(x_i)) - (1 - f_B(x_i))|^2 + |\Pi_A(x_i) - \Pi_B(x_i)|^2)}$
4. Normalized Euclidean distance N_{PVS}(A, B)

$$= \sqrt{\frac{1}{2n}\sum_{i=1}^{n}(|t_A(x_i) - t_B(x_i)|^2 + |(1 - f_A(x_i)) - (1 - f_B(x_i))|^2 + |\Pi_A(x_i) - \Pi_B(x_i)|^2)}$$

Definition 2.6.[11]

Distance measures for pythagorean vague sets [PVS] satisfies the following:

(i) 0 ⩽ Hamming Distance ⩽ 2n

(ii) 0 ⩽ Normalized Hamming Distance ⩽ 2

(iii) 0 ⩽ Euclidean Distance ⩽ √ 2n

(iv) 0 ⩽ Normalized Euclidean Distance ⩽ √ 2

where n = cardinality of the universal set

Definition 2.7.[11] (Union, Intersection and Complement of PVS)

Let P = {⟨x, (a, b)⟩ / x ∈ X}, P_1 = {⟨x, (a_1, b_1)⟩ /x ∈ X},P_2 = {⟨x, (a_2, b_2)⟩ / x ∈ X} be the Pythagorean vague sets and λ ⩾ 0 then (i) $P_1 \cup P_2$ = (max(a_1, a_2), max(b_1, b_2)) (ii) $P_1 \cap P_2$ = (min(a_1, a_2), min(b_1, b_2)) (iii) P^c = (b, a)

Definition 2.8. [13] (q-rung orthopair fuzzy set) Let X be a universe of discourse. A q-rung orthopair fuzzy set A (q-ROFS) in X is given by A ={⟨x, μ_A(x), ν_A(x)⟩ /x ∈ X}, where μ_A : X → [0,1] denotes the degree of membership and ν_A : X → [0,1] the degree of non-membership, of the element x∈X to the set A with the condition that $0 \leqslant (\mu_A(x))^q + (\nu_A(x))^q \leqslant 1$ (q ⩾ 1).

The degree of indeterminacy $\pi_A(x) = [1 - (\pi_A(x))^q - (\nu_A(x))^q)]^{\frac{1}{q}}$.
p = (μ, ν) is called a q-rung orthopair fuzzy number (q-ROFN). Set of all q-ROFN's defined on X is denoted as q-ROFN(X)

III. PYTHAGOREAN VAGUE BINARY SOFT SETS [PVBSS'S]

In this section Pythagorean vague binary soft set is developed with some of its basic notions.

Definition 3.1. (Pythagorean vague binary soft set) Let U_1, U_2 be two initial universes which is common to a fixed set A ⊆ E of parameters. Let PV(U_1) and PV(U_2) denote the power set of pythagorean vague subsets on U_1, U_2 respectively. A pair ($\ddot{F}$, A) is said to be a Pythagorean vague binary soft set [PVBSS]over U_1, U_2 where $\ddot{F}$ is a mapping given by $\ddot{F}$: A →PV (U_1)×PV (U_2) and ($\ddot{F}$, A) = $\{e_i \in A/(e_i, \ddot{F}(e_i))\}$; $\ddot{F}(e_i)$= $\left(\langle\frac{V_{\ddot{F}(e_i)}(x_j)}{x_j}; \forall e_i \in A, \forall x_j \in U_1\rangle, \langle\frac{V_{\ddot{F}(e_i)}(y_k)}{y_k}; \forall e_i \in A, \forall y_k \in U_2\rangle\right)$; where

$t_{\ddot{F}(e_i)}(x_j)$: U_1 → [0, 1] ; 1− $f_{\ddot{F}(e_i)}(x_j)$: U_1→[0, 1] ; $0 \leqslant [t_{\ddot{F}(e_i)}(x_j)]^2 + [1 - f_{\ddot{F}(e_i)}(x_j)]^2 \leqslant 1$ and

$t_{\ddot{F}(e_i)}(y_k)$: U_2 → [0, 1] ; 1− $f_{\ddot{F}(e_i)}(y_k)$: U_2→[0, 1] ; $0 \leqslant [t_{\ddot{F}(e_i)}(y_k)]^2 + [1 - f_{\ddot{F}(e_i)}(y_k)]^2 \leqslant 1$

Example 3.2. (Example for PVBSS) Let U_1= {f₁, f₂, f₃}, U_2= {b₁, b₂, b₃} be the set of flights and trains from Kochi to Bangalore respectively, and A = {e_1= first class, e_2= second class, e_3= third class} be the set of parameters with A ⊆ E. Let ($\ddot{F}$, A) be a PVBSS which describes the availability of tickets as follows:

$$(\ddot{F}, A) = \left\{\begin{array}{l} \left(e_1, \left(\langle\frac{[0.2,0.3]}{f_1}, \frac{[0.4,0.2]}{f_2}, \frac{[0.7,0.8]}{f_3}\rangle, \langle\frac{[0.3,0.8]}{b_1}, \frac{[0.6,0.7]}{b_2}, \frac{[0.4,0.1]}{b_3}\rangle\right)\right), \\ \left(e_2, \left(\langle\frac{[0.2,0.4]}{f_1}, \frac{[0.4,0.3]}{f_2}, \frac{[0.7,0.8]}{f_3}\rangle, \langle\frac{[0.3,0.2]}{b_1}, \frac{[0.6,0.7]}{b_2}, \frac{[0.4,0.6]}{b_3}\rangle\right)\right), \\ \left(e_3, \left(\langle\frac{[0.2,0.1]}{f_1}, \frac{[0.4,0.7]}{f_2}, \frac{[0.7,0.8]}{f_3}\rangle, \langle\frac{[0.3,0.8]}{b_1}, \frac{[0.6,0.7]}{b_2}, \frac{[0.4,0.6]}{b_3}\rangle\right)\right) \end{array}\right\}$$

Example 3.3. (Example for PVSS)

Let U= {u_1, u_2, u_3} be the universal set, and E={e_1, e_2, e_3} be the set of parameters with A ={e_1, e_3}⊆ E,

$$(F, A) = \left\{\left(e_1, \left(\frac{[0.2,0.3]}{f_1}, \frac{[0.4,0.2]}{f_2}, \frac{[0.7,0.8]}{f_3}\right)\right), \left(e_1, \left(\frac{[0.2,0.4]}{f_1}, \frac{[0.4,0.3]}{f_2}, \frac{[0.7,0.8]}{f_3}\right)\right), \left(e_1, \left(\frac{[0.2,0.1]}{f_1}, \frac{[0.4,0.7]}{f_2}, \frac{[0.7,0.8]}{f_3}\right)\right)\right\}$$

is a Pythagorean vague soft set.

Remark 3.4: Set of all Pythagorean vague soft sets on a common universe U is denoted by PVSS(U). Set of all Pythagorean vague binary soft sets over a common universe U_1, U_2 is denoted by PVBSS(U_1, U_2)

Remark 3.5: A comparison of properties of PVBSS's with different sets are given in table below:

IFS	PFS	PVS	PVBSS
$\mu_A(x)$: X → [0, 1]; $\nu_A(x)$: X → [0, 1]	$\mu_A(x)$: X → [0, 1]; $\nu_A(x)$: X → [0, 1]	$t_A(x)$: X → [0,1]; $[1-f_A(x)]$: X → [0,1]	$t_{\ddot{F}(e_i)}(x_j) : U_1$ → [0,1]; $1- f_{\ddot{F}(e_i)}(x_j): U_1$→ [0,1]; $t_{\ddot{F}(e_i)}(y_k) : U_2$ → [0,1]; $[1 - f_{\ddot{F}(e_i)}(y_k)]: U_2$ → [0,1]
$0 \leqslant \mu_A(x) + \nu_A(x) \leqslant 1$	$0 \leqslant [\mu_A(x)]^2 + [\nu_A(x)]^2 \leqslant 1$	$0 \leqslant [t_A(x)]^2 + [1-f_A(x)]^2 \leqslant 1$	$0 \leqslant [t_{\ddot{F}(e_i)}(x_j)]^2 + [1- f_{\ddot{F}(e_i)}(x_j)]^2 \leqslant 1$; $0 \leqslant [t_{\ddot{F}(e_i)}(y_k)]^2 + [1 - f_{\ddot{F}(e_i)}(y_k)]^2 \leqslant 1$
$\mu_A(x) + \nu_A(x) \leqslant 1$	$\mu_A(x) + \nu_A(x) \leqslant 1$; $\mu_A(x) + \nu_A(x) \geqslant 1$	$[t_A(x) + (1-f_A(x)] \leqslant 1$; $[t_A(x) + (1-f_A(x))] \geqslant 1$	$t_{\ddot{F}(e_i)}(x_j) + 1- f_{\ddot{F}(e_i)}(x_j) \leqslant 1$ or $t_{\ddot{F}(e_i)}(x_j) + 1- f_{\ddot{F}(e_i)}(x_j) \geqslant 1$ $t_{\ddot{F}(e_i)}(y_k) + (1 - f_{\ddot{F}(e_i)}(y_k)) \leqslant 1$ or $t_{\ddot{F}(e_i)}(y_k) + (1 - f_{\ddot{F}(e_i)}(y_k)) \geqslant 1$
$\mu_A(x) + \nu_A(x) + \Pi_A(x) = 1$	$[\mu_A(x)]^2 + [\nu_A(x)]^2 + [\Pi_A(x)]^2 = 1$	$[t_A(x)]^2 + [1-f_A(x)]^2 + [\Pi_A(x)]^2 = 1$	$[t_{\ddot{F}(e_i)}(x_j)]^2 + [1- f_{\ddot{F}(e_i)}(x_j)]^2 + [\Pi_{\ddot{F}(e_i)}(x_j)]^2 = 1$; $[t_{\ddot{F}(e_i)}(y_k)]^2 + [1 - f_{\ddot{F}(e_i)}(y_k)]^2 + [\Pi_{\ddot{F}(e_i)}(y_k)]^2 = 1$

IV. Operations on Pythagorean Vague binary soft sets

In this section some operations for PVBSS's are developed. Some classical set theoretical laws are also verified. For that consider two pythagorean vague binary soft sets as follows:

Let $(\ddot{F}, A) = \left\{\left(e_i, \left(\langle \frac{[t_{\ddot{F}(e_i)}(x_j),\ 1-f_{\ddot{F}(e_i)}(x_j)]}{x_j}; \forall e_i \in A, \forall x_j \in U_1\rangle, \langle \frac{[t_{\ddot{F}(e_i)}(y_k),\ 1-f_{\ddot{F}(e_i)}(y_k)]}{y_k}; \forall e_i \in A, \forall y_k \in U_2\rangle\right)\right)\right\}$,

and $(\ddot{G}, A) = \left\{\left(e_i, \left(\langle \frac{[t_{\ddot{G}(e_i)}(x_j),\ 1-f_{\ddot{G}(e_i)}(x_j)]}{x_j}; \forall e_i \in A, \forall x_j \in U_1\rangle, \langle \frac{[t_{\ddot{G}(e_i)}(y_k),\ 1-f_{\ddot{G}(e_i)}(y_k)]}{y_k}; \forall e_i \in A, \forall y_k \in U_2\rangle\right)\right)\right\}$

Definition 4.1. (Union and Intersection for PVBSS's)

(i) $(\ddot{F}, A) \cup (\ddot{G}, A) =$

$$\left\{\left(e_i, \left(\langle \frac{\max\left(t_{\ddot{F}(e_i)}(x_j), t_{\ddot{G}(e_i)}(x_j)\right),\ \max\left(1-f_{\ddot{F}(e_i)}(x_j),\ 1-f_{\ddot{G}(e_i)}(x_j)\right)}{x_j}\rangle \langle \frac{\max\left(t_{\ddot{F}(e_i)}(y_k), t_{\ddot{G}(e_i)}(y_k)\right),\ \max\left(1-f_{\ddot{F}(e_i)}(y_k),\ 1-f_{\ddot{G}(e_i)}(y_k)\right)}{y_k}\rangle\right)\right)\right\}$$

(ii) $(\ddot{F}, A) \cap (\ddot{G}, A) =$

$$\left\{\left(e_i, \left(\langle \frac{\min\left(t_{\ddot{F}(e_i)}(x_j), t_{\ddot{G}(e_i)}(x_j)\right),\ \min\left(1-f_{\ddot{F}(e_i)}(x_j),\ 1-f_{\ddot{G}(e_i)}(x_j)\right)}{x_j}\rangle \langle \frac{\min\left(t_{\ddot{F}(e_i)}(y_k),\ t_{\ddot{G}(e_i)}(y_k)\right),\ \min\left(1-f_{\ddot{F}(e_i)}(y_k), 1-f_{\ddot{G}(e_i)}(y_k)\right)}{y_k}\rangle\right)\right)\right\}$$

; $\forall e_i \in A, \forall x_j \in U_1$ and $\forall y_k \in U_2$

Definition 4.2. (Complement of a PVBSS)

$(\ddot{F}, A)^c =$

$$\left\{\left(e_i, \left(\langle \frac{[1-f_{\ddot{F}(e_i)}(x_j),\ t_{\ddot{F}(e_i)}(x_j)]}{x_j}; \forall e_i \in A, \forall x_j \in U_1\rangle \langle \frac{[1-f_{\ddot{F}(e_i)}(y_k),\ t_{\ddot{F}(e_i)}(y_k)]}{y_k}; \forall e_i \in A, \forall y_k \in U_2\rangle\right)\right)\right\}$$

; where $\forall e_i \in A, \forall x_j \in U_1$ and $\forall y_k \in U_2$

Definition 4.3. (Sum and Product of two PVBSS's)

Let $(\ddot{F}, A), (\ddot{G}, A) \in$ PVBSS(U_1, U_2)

(i) Sum of two pythagorean vague binary soft sets

$(\ddot{F}, A) \oplus$pvbss $(\ddot{G}, A) =$

$$\left\{\langle \frac{\sqrt{\left(t_{\ddot{F}(e_i)}(x_j)\right)^2+\left(t_{\ddot{G}(e_i)}(x_j)\right)^2-\left(t_{\ddot{F}(e_i)}(x_j)\right)^2.\left(t_{\ddot{G}(e_i)}(x_j)\right)^2},\ \left(1-f_{\ddot{F}(e_i)}(x_j),\ 1-f_{\ddot{G}(e_i)}(x_j)\right)}{x_j}; \forall e_i \in A, \forall x_j \in U_1\rangle\right\} +$$

$$\left\{\langle \frac{\sqrt{\left(t_{\ddot{F}(e_i)}(y_k)\right)^2+\left(t_{\ddot{G}(e_i)}(y_k)\right)^2-\left(t_{\ddot{F}(e_i)}(y_k)\right)^2.\left(t_{\ddot{G}(e_i)}(y_k)\right)^2},\ \left(1-f_{\ddot{F}(e_i)}(y_k),\ 1-f_{\ddot{G}(e_i)}(y_k)\right)}{y_k}; \forall e_i \in A, \forall y_k \in U_2\rangle\right\}$$

where $\forall e_i \in A, \forall x_j \in U_1$ and $\forall y_k \in U_2$

(ii) Product of two pythagorean vague binary soft sets

$(\ddot{F}, A) \otimes$pvbss $(\ddot{G}, A) =$

$$\left\{\langle \frac{\sqrt{\left(1-f_{\ddot{F}(e_i)}(x_j)\right)^2+\left(1-f_{\ddot{G}(e_i)}(x_j)\right)^2-\left(1-f_{\ddot{F}(e_i)}(x_j)\right)^2.\left(1-f_{\ddot{G}(e_i)}(x_j)\right)^2},\ \left(t_{\ddot{F}(e_i)}(x_j),\ t_{\ddot{G}(e_i)}(x_j)\right)}{x_j}; \forall e_i \in A, \forall x_j \in U_1\rangle\right\} +$$

$$\left\{\langle \frac{\sqrt{\left(1-f_{\ddot{F}(e_i)}(y_k)\right)^2+\left(1-f_{\ddot{G}(e_i)}(y_k)\right)^2-\left(1-f_{\ddot{F}(e_i)}(y_k)\right)^2.\left(1-f_{\ddot{G}(e_i)}(y_k)\right)^2},\ \left(t_{\ddot{F}(e_i)}(y_k),\ t_{\ddot{G}(e_i)}(y_k)\right)}{y_k}; \forall e_i \in A, \forall y_k \in U_2\rangle\right\} +$$

where $\forall e_i \in$ A, $\forall x_j \in U_1$ and $\forall y_k \in U_2$

Definition 4.4. (Properties of PVBSS's)

Let (F, A), (G, A) ∈ PVBSS (U_1, U_2). Following properties found true:

(i) Idempotent Laws

Operations union, intersection, sum and product follows idempotent laws for PVBSS's

(1) ($\ddot{F}$, A) ∩ ($\ddot{F}$, A) = ($\ddot{F}$, A)

(2) ($\ddot{F}$, A) ∪ ($\ddot{F}$, A) = ($\ddot{F}$, A)

(3) ($\ddot{F}$, A) $\oplus_{pvbss}$ ($\ddot{F}$, A) = ($\ddot{F}$, A)

(4) ($\ddot{F}$, A) $\otimes_{pvbss}$ ($\ddot{F}$, A) = ($\ddot{F}$, A)

(ii) Commutativity

Operations union, intersection, sum and product follows commutativity for PVBSS's

(1) ($\ddot{F}$, A) ∩ ($\ddot{G}$, A) = ($\ddot{G}$, A) ∩ ($\ddot{G}$, A)

(2) ($\ddot{F}$, A) ∪ ($\ddot{G}$, A) = ($\ddot{G}$, A) ∪ ($\ddot{G}$, A)

(3) ($\ddot{F}$, A) $\oplus_{pvbss}$ ($\ddot{G}$, A) = ($\ddot{G}$, A) $\oplus_{pvbss}$ ($\ddot{F}$, A)

(4) ($\ddot{F}$, A) $\otimes_{pvbss}$ ($\ddot{G}$, A) = ($\ddot{G}$, A) $\otimes_{pvbss}$ $\ddot{F}$, A)

(iii) De Morgan's Laws

Union and intersection violates De Morgan's laws. But sum and product operations follows

(1)(($\ddot{F}$, A) ∩ ($\ddot{G}$, A))^c ≠ ($\ddot{F}$, A))^c ∪ ($\ddot{G}$, A))^c. But (($\ddot{F}$, A) ∩ ($\ddot{G}$, A))^c = ($\ddot{F}$, A))^c ∩ ($\ddot{G}$, A))^c

(2)(($\ddot{F}$, A) ∪ ($\ddot{G}$, A))^c ≠ ($\ddot{F}$, A))^c ∩ ($\ddot{G}$, A))^c. But (($\ddot{F}$, A) ∪ ($\ddot{G}$, A))^c = ($\ddot{F}$, A))^c ∪ ($\ddot{G}$, A))^c

(3)(($\ddot{F}$, A) $\oplus_{pvbss}$ ($\ddot{G}$, A))^c = ($\ddot{F}$, A))^c $\otimes_{pvbss}$ ($\ddot{G}$, A))^c

(4)(($\ddot{F}$, A) $\otimes_{pvbss}$ ($\ddot{G}$, A))^c = ($\ddot{F}$, A))^c $\oplus_{pvbss}$ ($\ddot{G}$, A))^c

V. VARIOUS DISTANCE MEASURES FOR PVBSS

In this section various distance measures for PVBSS's are developed. For that consider two PVBSS's as follows:

Let ($\ddot{F}$, A) = $\left\{\left(e_i, \left(\left\langle \frac{\left[t_{\ddot{F}(e_i)}(x_j),\ 1-f_{\ddot{F}(e_i)}(x_j)\right]}{x_j}; \forall e_i \in A, \forall x_j \in U_1\right\rangle, \left\langle \frac{\left[t_{\ddot{F}(e_i)}(y_k),\ 1-f_{\ddot{F}(e_i)}(y_k)\right]}{y_k}; \forall e_i \in A, \forall y_k \in U_2\right\rangle\right)\right)\right\}$,

and ($\ddot{G}$, A) = $\left\{\left(e_i, \left(\left\langle \frac{\left[t_{\ddot{G}(e_i)}(x_j),\ 1-f_{\ddot{G}(e_i)}(x_j)\right]}{x_j}; \forall e_i \in A, \forall x_j \in U_1\right\rangle, \left\langle \frac{\left[t_{\ddot{G}(e_i)}(y_k),\ 1-f_{\ddot{G}(e_i)}(y_k)\right]}{y_k}; \forall e_i \in A, \forall y_k \in U_2\right\rangle\right)\right)\right\}$

Definition 5.1. Let ($\ddot{F}$, A) and ($\ddot{G}$, A) be two PVBSS's over the common universe U_1, U_2 and let A ⊆ E be the set of parameters. Then various distance measures can be formulated from the following formulae :

(i) $d_{pvbss}(\ddot{F}, \ddot{G})$ =

$$\frac{1}{4m}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\left|t_{\ddot{F}(e_i)}(x_j)-t_{\ddot{G}(e_i)}(x_j)\right|^q+\left|\left(1-f_{\ddot{F}(e_i)}(x_j)\right)-\left(1-f_{\ddot{G}(e_i)}(x_j)\right)\right|^q+\left|\pi_{\ddot{F}(e_i)}(x_j)-\pi_{\ddot{G}(e_i)}(x_j)\right|^q\right]+$$
$$\frac{1}{4m}\sum_{i=1}^{m}\sum_{k=1}^{p}\left[\left|t_{\ddot{F}(e_i)}(y_k)-t_{\ddot{G}(e_i)}(y_k)\right|^q+\left|\left(1-f_{\ddot{F}(e_i)}(y_k)\right)-\left(1-f_{\ddot{G}(e_i)}(y_k)\right)\right|^q+\left|\pi_{\ddot{F}(e_i)}(y_k)-\pi_{\ddot{G}(e_i)}(y_k)\right|^q\right]$$

(ii) $d_{pvbss}(\ddot{F}, \ddot{G})$ =

$$\sqrt[q]{\begin{array}{l}\frac{1}{4mn}\sum_{i=1}^{m}\sum_{j=i}^{n}\left[\left|t_{\ddot{F}(e_i)}(x_j)-t_{\ddot{G}(e_i)}(x_j)\right|^q+\left|\left(1-f_{\ddot{F}(e_i)}(x_j)\right)-\left(1-f_{\ddot{G}(e_i)}(x_j)\right)\right|^q+\left|\pi_{\ddot{F}(e_i)}(x_j)-\pi_{\ddot{G}(e_i)}(x_j)\right|^q\right]+\\ \frac{1}{4mp}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\left|t_{\ddot{F}(e_i)}(x_j)-t_{\ddot{G}(e_i)}(x_j)\right|^q+\left|\left(1-f_{\ddot{F}(e_i)}(x_j)\right)-\left(1-f_{\ddot{G}(e_i)}(x_j)\right)\right|^q+\left|\pi_{\ddot{F}(e_i)}(x_j)-\pi_{\ddot{G}(e_i)}(x_j)\right|^q\right]\end{array}}$$

where q is always a positive integer.

Case(1) : If q = 1 then (i) and (ii) reduced to hamming distance [denoted as $d^{H}_{pvbss}(\ddot{F}, \ddot{G})$] and normalised hamming distance [denoted as $d^{nH}_{pvbss}(\ddot{F}, \ddot{G})$] respectively

Case (2) : If q = 2 then (i) and (ii) reduced to euclidean distance[denoted as $d^{E}_{pvbss}(\ddot{F}, \ddot{G})$] and normalised euclidean distance [denoted as $d^{nE}_{pvbss}(\ddot{F}, \ddot{G})$] respectively

Distance measures for PVBSS's satisfies the following:

(i) $0 \leqslant d^{H}_{pvbss}(\ddot{F}, \ddot{G}) \leqslant 2\,[\#(U_1)+\#(U_2)]$
(ii) $0 \leqslant d^{nH}_{pvbss}(\ddot{F}, \ddot{G}) \leqslant 2$
(iii) $0 \leqslant d^{E}_{pvbss}(\ddot{F}, \ddot{G}) \leqslant \sqrt{2[\#(U_1)+\#(U_2)]}$
(iv) $0 \leqslant d^{nE}_{pvbss}(\ddot{F}, \ddot{G}) \leqslant 2$

\# denotes the cardinality of the set

Remark : It is observed that,
(1) $d^{nE}_{pvbss} \leqslant d^{nH}_{pvbss} \leqslant d^{E}_{pvbss} \leqslant d^{H}_{pvbss}$
(2) Calculations showed that d^{nH}_{pvbss} and d^{nE}_{pvbss} are more accurate than the others.

VI. ENTROPY OF PVBSS'S

In this section entropy of PVBSS's with it's definition and properties are discussed.

Definition 6.1. Let H: PVBSS(U_1)× PVBSS(U_2)→[0, 1] be a mapping. PVBSS(U_1, U_2) be the set of all PVBSS's over the common universe {U_1, U_2} where $U_1 = \{x_1, x_2, -----, x_j\}$ and $U_2 = \{y_1, y_2, -----, y_k\}$. For $(\ddot{P}, A) \in$ PVBSS(U_1) × PVBSS(U_2) with $A = \{e_1, e_2, -----, e_m\} \subseteq E$ the set of parameters then H($\ddot{P}$, A) is called the entropy of ($\ddot{P}$, A) if it satisfies the following conditions:

(i) $H(\ddot{P}, A) = 0 \Leftrightarrow \forall e_i \in A, \forall x_j \in U_1,$
$t_{\ddot{P}(e_i)}(x_j) = 0,\ (1-f_{\ddot{P}(e_i)}(x_j)) = 1$ or $t_{\ddot{P}(e_i)}(x_j) = 1,\ (1-f_{\ddot{P}(e_i)}(x_j)) = 0;$
and $\forall e_i \in A, \forall y_k \in U_2,$
$t_{\ddot{P}(e_i)}(y_k) = 0,\ (1-f_{\ddot{P}(e_i)}(y_k)) = 1$ or $t_{\ddot{P}(e_i)}(y_k) = 1,\ (1-f_{\ddot{P}(e_i)}(y_k)) = 0;$

(ii) $H(\ddot{P}, A) = 1 \Leftrightarrow n = p;$
$\forall e_i \in A, \forall x_j \in U_1, t_{\ddot{P}(e_i)}(x_j) = (1-f_{\ddot{P}(e_i)}(x_j));$
$\forall e_i \in A, \forall y_k \in U_2, t_{\ddot{P}(e_i)}(y_k) = (1-f_{\ddot{P}(e_i)}(y_k))$

(iii) $H(\ddot{P}, A) = H(\ddot{P}, A)^c$

(iv) $\forall e_i \in A, \forall x_j \in U_1$, and $\forall y_k \in U_2$,

(1) If $(\ddot{P}, A) \subseteq (\ddot{R}, A)$; $t_{\ddot{R}(e_i)}(x_j) \leqslant (1-f_{\ddot{R}(e_i)}(x_j))$; $t_{\ddot{P}(e_i)}(y_k) \leqslant (1-f_{\ddot{R}(e_i)}(y_k))$ then $H(\ddot{P}, A) \leqslant H(\ddot{R}, A)$
(2) If $(\ddot{P}, A) \supseteq (\ddot{R}, A)$; $t_{\ddot{R}(e_i)}(x_j) \geqslant (1-f_{\ddot{R}(e_i)}(x_j))$ and $t_{\ddot{R}(e_i)}(y_k) \geqslant (1-f_{\ddot{R}(e_i)}(y_k))$ then $H(\ddot{P}, A) \geqslant H(\ddot{R}, A)$

Theorem 6.2. Let $U_1 = \{x_1, x_2, -----, x_j\}$ and $U_2 = \{y_1, y_2, -----, y_k\}$ be the common universe and $A = \{e_1, e_2, -----, e_m\}$ be the universal set of parameters. Let ($\ddot{P}$, A) be a family of all PVBSS's. Define H ($\ddot{P}$, A) as follows: $H(\ddot{P}, A) = 1 \Leftrightarrow \frac{\sum_{t=1}^{w} H_t(\ddot{P}, A)}{w}$

where $H_t(\ddot{P}, A) = \frac{1}{4mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{\left|1-t^2{}_{\ddot{F}(e_i)}(x_j)-\left(1-f^2{}_{\ddot{F}(e_i)}(x_j)\right)\right|}{\left|1+t^2{}_{\ddot{F}(e_i)}(x_j)-\left(1-f^2{}_{\ddot{F}(e_i)}(x_j)\right)\right|} + \frac{1}{4mp}\sum_{i=1}^{m}\sum_{k=1}^{p}\frac{\left|1-t^2{}_{\ddot{F}(e_i)}(y_k)-\left(1-f^2{}_{\ddot{F}(e_i)}(y_k)\right)\right|}{\left|1+t^2{}_{\ddot{F}(e_i)}(y_k)-\left(1-f^2{}_{\ddot{F}(e_i)}(y_k)\right)\right|} = 1$

Then H ($\ddot{P}$, A) is an entropy for PVBSS's

Proof (i) H ($\ddot{P}$, A) = 0 $\Leftrightarrow \sum_{t=1}^{w} H_t(\ddot{P}, \text{A}) = 0$

$\Leftrightarrow 1-\left|t^2{}_{\ddot{P}(e_i)}(x_j) - \left(1 - f^2{}_{\ddot{P}(e_i)}(x_j)\right)\right|=0$; for $0 \leqslant t^2{}_{\ddot{P}(e_i)}(x_j) \leqslant 1$ and $0 \leqslant \left(1 - f^2{}_{\ddot{P}(e_i)}(x_j)\right) \leqslant 1$

and $1 - \left|t^2{}_{\ddot{P}(e_i)}(y_k) - \left(1 - f^2{}_{\ddot{P}(e_i)}(y_k)\right)\right|=0$; for $0 \leqslant t^2{}_{\ddot{P}(e_i)}(y_k) \leqslant 1$ and $0 \leqslant \left(1 - f^2{}_{\ddot{P}(e_i)}(y_k)\right) \leqslant 1$

$\left|t^2{}_{\ddot{P}(e_i)}(x_j) - \left(1 - f^2{}_{\ddot{P}(e_i)}(x_j)\right)\right| \leqslant 1 \Rightarrow \left|t^2{}_{\ddot{P}(e_i)}(x_j) - \left(1 - f^2{}_{\ddot{P}(e_i)}(x_j)\right)\right|=1.$

$\left|t^2{}_{\ddot{P}(e_i)}(y_k) - \left(1 - f^2{}_{\ddot{P}(e_i)}(y_k)\right)\right| \leqslant 1 \Rightarrow \left|t^2{}_{\ddot{P}(e_i)}(y_k) - \left(1 - f^2{}_{\ddot{P}(e_i)}(y_k)\right)\right|=1.$

$\therefore \forall\, e_i \in$ A, $x_j \in \text{U}_1$, $t_{\ddot{P}(e_i)}(x_j) = 0$, $(1-f_{\ddot{P}(e_i)}(x_j)) = 1$ or $t_{\ddot{P}(e_i)}(x_j) = 1$, $(1-f_{\ddot{P}(e_i)}(x_j)) = 0$ and

$\therefore \forall\, e_i \in$ A, $y_k \in \text{U}_2$, $t_{\ddot{P}(e_i)}(y_k) = 0$, $(1-f_{\ddot{P}(e_i)}(y_k)) = 1$ or $t_{\ddot{P}(e_i)}(y_k) = 1$, $(1-f_{\ddot{P}(e_i)}(y_k)) = 0$

(ii) H($\ddot{P}$, A) = 1 $\Leftrightarrow \sum_{t=1}^{w} H_t(\ddot{P}, \text{A}) = \text{w} \Leftrightarrow H_t(\ddot{P}, \text{A}) = 1$

$$\Leftrightarrow \frac{1}{4mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{1-\left|t^2{}_{\ddot{P}(e_i)}(x_j)-\left(1-f^2{}_{\ddot{P}(e_i)}(x_j)\right)\right|}{1+\left|t^2{}_{\ddot{P}(e_i)}(x_j)-\left(1-f^2{}_{\ddot{P}(e_i)}(x_j)\right)\right|} + \frac{1}{4mp}\sum_{i=1}^{m}\sum_{k=1}^{p}\frac{1-\left|t^2{}_{\ddot{P}(e_i)}(y_k)-\left(1-f^2{}_{\ddot{P}(e_i)}(y_k)\right)\right|}{1+\left|t^2{}_{\ddot{P}(e_i)}(y_k)-\left(1-f^2{}_{\ddot{P}(e_i)}(y_k)\right)\right|} = 1$$

$$\Leftrightarrow \frac{1}{4mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{1-\left|t^2{}_{\ddot{P}(e_i)}(x_j)-\left(1-f^2{}_{\ddot{P}(e_i)}(x_j)\right)\right|}{1+\left|t^2{}_{\ddot{P}(e_i)}(x_j)-\left(1-f^2{}_{\ddot{P}(e_i)}(x_j)\right)\right|} = \frac{1}{2} = \frac{1}{4mp}\sum_{i=1}^{m}\sum_{k=1}^{p}\frac{1-\left|t^2{}_{\ddot{P}(e_i)}(y_k)-\left(1-f^2{}_{\ddot{P}(e_i)}(y_k)\right)\right|}{1+\left|t^2{}_{\ddot{P}(e_i)}(y_k)-\left(1-f^2{}_{\ddot{P}(e_i)}(y_k)\right)\right|}$$

$\Leftrightarrow$ n = p and

$1 - \left|t^2{}_{\ddot{P}(e_i)}(x_j) - \left(1 - f^2{}_{\ddot{P}(e_i)}(x_j)\right)\right|=1 + \left|t^2{}_{\ddot{P}(e_i)}(x_j) - \left(1 - f^2{}_{\ddot{P}(e_i)}(x_j)\right)\right|$ and

$1 - \left|t^2{}_{\ddot{P}(e_i)}(y_k) - \left(1 - f^2{}_{\ddot{P}(e_i)}(y_k)\right)\right|=1 + \left|t^2{}_{\ddot{P}(e_i)}(y_k) - \left(1 - f^2{}_{\ddot{P}(e_i)}(y_k)\right)\right|$

$\Leftrightarrow$ n = p and

$\left|t^2{}_{\ddot{P}(e_i)}(x_j) - \left(1 - f^2{}_{\ddot{P}(e_i)}(x_j)\right)\right|= 0$; $\forall\, e_i \in$ A, $x_j \in \text{U}_1$ and

$\left|t^2{}_{\ddot{P}(e_i)}(y_k) - \left(1 - f^2{}_{\ddot{P}(e_i)}(y_k)\right)\right|= 0$; $\forall\, e_i \in$ A, $y_k \in \text{U}_2$

$\Leftrightarrow$ n = p and

$\text{t}_{\ddot{P}(e_i)}(x_j) = \left(1 - \text{f}_{\ddot{P}(e_i)}(x_j)\right) = 0$; $\forall\, e_i \in$ A, $x_j \in \text{U}_1$ and

$\text{t}_{\ddot{P}(e_i)}(y_k) = \left(1 - f_{\ddot{P}(e_i)}(y_k)\right) = 0$; $\forall\, e_i \in$ A, $y_k \in \text{U}_2$

(iii) $H_i(\ddot{P}, \text{A})$

$$= \frac{1}{4mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{1-\left|t^2{}_{\ddot{P}(e_i)}(x_j)-\left(1-f^2{}_{\ddot{P}(e_i)}(x_j)\right)\right|}{1+\left|t^2{}_{\ddot{P}(e_i)}(x_j)-\left(1-f^2{}_{\ddot{P}(e_i)}(x_j)\right)\right|} + \frac{1}{4mp}\sum_{i=1}^{m}\sum_{k=1}^{p}\frac{1-\left|t^2{}_{\ddot{P}(e_i)}(y_k)-\left(1-f^2{}_{\ddot{P}(e_i)}(y_k)\right)\right|}{1+\left|t^2{}_{\ddot{P}(e_i)}(y_k)-\left(1-f^2{}_{\ddot{P}(e_i)}(y_k)\right)\right|}$$

$$= \frac{1}{4mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{1-\left|\left(1-f^2{}_{\ddot{P}^c(\neg e_i)}(x_j)\right)-t^2{}_{\ddot{P}^c(\neg e_i)}(x_j)\right|}{1+\left|\left(1-f^2{}_{\ddot{P}^c(\neg e_i)}(x_j)\right)-t^2{}_{\ddot{P}^c(\neg e_i)}(x_j)\right|} + \frac{1}{4mp}\sum_{i=1}^{m}\sum_{k=1}^{p}\frac{1-\left|\left(1-f^2{}_{\ddot{P}^c(\neg e_i)}(y_k)\right)-t^2{}_{\ddot{P}^c(\neg e_i)}(y_k)\right|}{1+\left|\left(1-f^2{}_{\ddot{P}^c(\neg e_i)}(y_k)\right)-t^2{}_{\ddot{P}^c(\neg e_i)}(y_k)\right|}$$

$$= \frac{1}{4mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{1-\left|t^2{}_{\ddot{P}^c(\neg e_i)}(x_j)-\left(1-f^2{}_{\ddot{P}^c(\neg e_i)}(x_j)\right)\right|}{1+\left|t^2{}_{\ddot{P}^c(\neg e_i)}(x_j)-\left(1-f^2{}_{\ddot{P}^c(\neg e_i)}(x_j)\right)\right|} + \frac{1}{4mp}\sum_{i=1}^{m}\sum_{k=1}^{p}\frac{1-\left|t^2{}_{\ddot{P}^c(\neg e_i)}(y_k)-\left(1-f^2{}_{\ddot{P}^c(\neg e_i)}(y_k)\right)\right|}{1+\left|t^2{}_{\ddot{P}^c(\neg e_i)}(y_k)-\left(1-f^2{}_{\ddot{P}^c(\neg e_i)}(y_k)\right)\right|}$$

$= H_i(\ddot{P}, \text{A})^c$

$\therefore$ H($\ddot{P}$, A) = H($\ddot{P}$, A)^c

(iv) (1) When ($\ddot{P}$, A) $\subseteq$ ($\ddot{R}$, A) and $\text{t}_{\ddot{R}(e_i)}(x_j) \leqslant (1-f_{\ddot{R}(e_i)}(\text{x}_j))$ for $\forall\, e_i \in$ A, $x_j \in \text{U}_1$ and
$\text{t}_{\ddot{F}(e_i)}(y_k) \leqslant \left(1 - f_{\ddot{F}(e_i)}(y_k)\right)$ for $\forall\, e_i \in$ A, $y_k \in \text{U}_2$

$\Rightarrow 0 \leqslant t_{\ddot{P}(e_i)}(x_j) \leqslant t_{\ddot{R}(e_i)}(x_j) \leqslant (1-f_{\ddot{R}(e_i)}(x_j)) \leqslant (1-f_{\ddot{P}(e_i)}(x_j)) \leqslant 1$ and
$0 \leqslant t_{\ddot{P}(e_i)}(y_k) \leqslant t_{\ddot{R}(e_i)}(y_k) \leqslant (1-f_{\ddot{R}(e_i)}(y_k)) \leqslant (1-f_{\ddot{P}(e_i)}(y_k)) \leqslant 1$

$\Rightarrow \left|t^2{}_{\ddot{P}(e_i)}(x_j) - \left(1 - f^2{}_{\ddot{P}(e_i)}(x_j)\right)\right| \geq \left|t^2{}_{\ddot{R}(e_i)}(x_j) - \left(1 - f^2{}_{\ddot{R}(e_i)}(x_j)\right)\right|$ and
$\left|t^2{}_{\ddot{P}(e_i)}(y_k) - (1 - f^2{}_{\ddot{P}(e_i)}(y_k))\right| \geq \left|t^2{}_{\ddot{R}(e_i)}(y_k) - (1 - f^2{}_{\ddot{R}(e_i)}(y_k))\right|$

$\Rightarrow 1-\left|t^2{}_{\ddot{P}(e_i)}(x_j) - \left(1 - f^2{}_{\ddot{P}(e_i)}(x_j)\right)\right| \leqslant 1-\left|t^2{}_{\ddot{R}(e_i)}(x_j) - \left(1 - f^2{}_{\ddot{R}(e_i)}(x_j)\right)\right|$,
$1+\left|t^2{}_{\ddot{P}(e_i)}(x_j) - \left(1 - f^2{}_{\ddot{P}(e_i)}(x_j)\right)\right| \geq 1+\left|t^2{}_{\ddot{R}(e_i)}(x_j) - \left(1 - f^2{}_{\ddot{R}(e_i)}(x_j)\right)\right|$ and
$1-\left|t^2{}_{\ddot{P}(e_i)}(y_k) - \left(1 - f^2{}_{\ddot{P}(e_i)}(y_k)\right)\right| \leq 1-\left|t^2{}_{\ddot{R}(e_i)}(y_k) - \left(1 - f^2{}_{\ddot{R}(e_i)}(y_k)\right)\right|$,
$1+\left|t^2{}_{\ddot{P}(e_i)}(y_k) - \left(1 - f^2{}_{\ddot{P}(e_i)}(y_k)\right)\right| \geq 1+\left|t^2{}_{\ddot{R}(e_i)}(y_k) - \left(1 - f^2{}_{\ddot{R}(e_i)}(y_k)\right)\right|$.

Thus $H_t(\ddot{P}, A) \leqslant H_t(\ddot{R}, A) \Rightarrow H(\ddot{P}, A) \leqslant H(\ddot{R}, A)$.

(2) Similarly when $(\ddot{P}, A) \supseteq (\ddot{R}, A)$ and $t_{\ddot{R}(e_i)}(x_j) \geq (1-f_{\ddot{R}(e_i)}(x_j))$ for $\forall\, e_i \in A$, $x_j \in U_1$ and
$t_{\ddot{R}(e_i)}(y_k) \geq (1-f_{\ddot{R}(e_i)}(y_k))$ for $\forall\, e_i \in A$, $y_k \in U_2$.
$H_t(\ddot{P}, A) \geq H_t(\ddot{R}, A) \Rightarrow H(\ddot{P}, A) \geq H(\ddot{R}, A)$

Theorem 6.3. Let $(\ddot{P}, A) \in PVBSS(U_1) \times PVBSS(U_2)$ and let $H(\ddot{P}, A)$ is an entropy on $PVBSS(U_1) \times PVBSS(U_2)$. If the function $g : [0,1] \to [0,1]$ be strictly monotone increasing real function and $g(0) = 0$, $g(1) = 1$ then $H'(\ddot{P}, A) = g(H(\ddot{P}, A))$ is also an entropy on $PVBSS(U_1) \times PVBSS(U_2)$

Proof (i) $H'(\ddot{P}, A) = 0 \Leftrightarrow g(H(\ddot{P}, A)) = 0 \Leftrightarrow H(\ddot{P}, A) = 0 \Leftrightarrow$
$\forall\, e_i \in A, x_j \in U_1$, $t_{\ddot{P}(e_i)}(x_j) = 0$, $\left(1 - f_{\ddot{P}(e_i)}(x_j)\right) = 1$ or $t_{\ddot{P}(e_i)}(x_j) = 1$, $\left(1 - f_{\ddot{P}(e_i)}(x_j)\right) = 0$ and
$\forall\, e_i \in A$, $y_k \in U_2$, $t_{\ddot{P}(e_i)}(y_k) = 0$, $\left(1 - f_{\ddot{P}(e_i)}(y_k)\right) = 1$ or $t_{\ddot{F}(e_i)}(y_k) = 1$, $\left(1 - f_{\ddot{F}(e_i)}(y_k)\right) = 0$

(ii) $H'(\ddot{P}, A) = 1 \Leftrightarrow g(H(\ddot{P}, A)) = 1 \Leftrightarrow H(\ddot{P}, A) = 1$
$\Leftrightarrow$ n = p and $\forall\, e_i \in A$, $x_j \in U_1$, $t_{\ddot{P}(e_i)}(x_j) = \left(1 - f_{\ddot{P}(e_i)}(x_j)\right)$, and
$\forall\, e_i \in A$, $y_k \in U_2$, $t_{\ddot{P}(e_i)}(y_k) = \left(1 - f_{\ddot{P}(e_i)}(y_k)\right)$

(iii) $H'(\ddot{P}, A)^c = (H'(\ddot{P}, A))^c = (g(H(\ddot{P}, A))^c = (g(H(\ddot{P}, A))^c) = g(H(\ddot{P}, A)) = H'(\ddot{P}, A)$

(iv)If $(\ddot{P}, A) \supset (\ddot{R}, A)$ then $H(\ddot{P}, A) \leqslant H(\ddot{R}, A)$. g is monotone increasing real function
$\Rightarrow H'(\ddot{P}, A) \leqslant g(H(\ddot{P}, A)) \leqslant g(H(\ddot{R}, A)) = H(\ddot{R}, A)$ $\therefore$ $(\ddot{P}, A) \supset (\ddot{R}, A) \Rightarrow H'(\ddot{P}, A) \leqslant H'(\ddot{R}, A)$.

Similarly, if $(\ddot{P}, A) \supset (\ddot{R}, A) \Rightarrow H(\ddot{P}, A) \geqslant H(\ddot{R}, A)$. Fourth condition also got satisfied. Hence if $H(\ddot{P}, A)$ is an entropy with a monotone increasing real function g with the given conditions then $g(H(\ddot{P}, A))$ is also an entropy.

Theorem 6.4 Let $\Re$ ={R/R is an entropy of vague soft sets}. If the function $h:[0,1]^n \to [0,1]$, $a = (a_1, a_2, \text{-----}, a_n) \to h(a)$ satisfies the following conditions:

(1) $h(a) = 0 \Leftrightarrow \frac{1}{n}\sum_{i=1}^{n} a_i = 0$
(2) $h(a) = 1 \Leftrightarrow \frac{1}{n}\sum_{i=1}^{n} a_i = 1$
(3) If $a \leqslant \hat{a}$, that is $a_i \leqslant \hat{a}$, $1 \leqslant i \leqslant n$ then $h(a) \leqslant h(\hat{a})$. Then $R_h(\ddot{K}, A) = h(R_1(\ddot{K}, A), R_2(\ddot{K}, A), \text{----------}, R_n(\ddot{K}, A))$, $R_i \in \Re$; $1 \leqslant i \leqslant n$ is an entropy on PVBSS's

Proof (i)Using condition (1),
$R_h(\ddot{K}, A) = h(R_1(\ddot{K}, A), R_2(\ddot{K}, A), \text{----------}, R_n(\ddot{K}, A)) = 0 \Leftrightarrow \frac{1}{n}\sum_{i=1}^{n} R_i(\ddot{K}, A) = 0 \Leftrightarrow R_i(\ddot{K}, A) = 0$; $1 \leqslant i \leqslant n$
$\Leftrightarrow \forall\, e_i \in A$, $x_j \in U_1$, $t_{\ddot{K}(e_i)}(x_j) = 0$, $\left(1 - f_{\ddot{K}(e_i)}(x_j)\right) = 1$ or $t_{\ddot{K}(e_i)}(x_j) = 1$, $\left(1 - f_{\ddot{K}(e_i)}(x_j)\right) = 0$ and
$\forall\, e_i \in A$, $y_k \in U_2$, $t_{\ddot{K}(e_i)}(y_k) = 0$, $\left(1 - f_{\ddot{K}(e_i)}(y_k)\right) = 1$ or $t_{\ddot{K}(e_i)}(y_k) = 1$, $\left(1 - f_{\ddot{K}(e_i)}(y_k)\right) = 0$

(ii) Using condition(2),
$R_h(\ddot{R}, A) = h(R_1(\ddot{R}, A), R_2(\ddot{R}, A), \text{----------}, R_n(\ddot{R}, A)) = 1 \Leftrightarrow \frac{1}{n}\sum_{i=1}^{n} R_i(\ddot{K}, A) = 1 \Leftrightarrow R_i(\ddot{K}, A) = 1$; $1 \leqslant i \leqslant n$
$\Leftrightarrow \forall\, e_i \in A$, $x_j \in U_1$, $t_{\ddot{K}(e_i)}(x_j) = \left(1 - f_{\ddot{K}(e_i)}(x_j)\right)$ and $\forall\, e_i \in A$, $y_k \in U_2$, $t_{\ddot{K}(e_i)}(y_k) = \left(1 - f_{\ddot{K}(e_i)}(y_k)\right)$

(iii) $R_h(\ddot{K}, A)^c = h(R_1(\ddot{K}, A)^c, R_2(\ddot{K}, A)^c, \text{----------}, R_n(\ddot{K}, A)^c) = h(R_1(\ddot{K}, A), R_2(\ddot{K}, A), \text{----------}, R_n(\ddot{K}, A)) = R_h(\ddot{K}, A)$

(iv) If R($\ddot{K}$, A)≤ R($\ddot{N}$, A) then R_i($\ddot{K}$, A) ≤ R_i($\ddot{N}$, A), ∀ $R_i \in \Re$. Using condition(3), R($\ddot{K}$, A) ≤ R($\ddot{N}$, A)
⇒ R_h($\ddot{K}$, A) = h(R_1($\ddot{R}$, A), R_2($\ddot{R}$, A),−−−−−−−−−−−, R_n($\ddot{R}$, A)) ≤ R_h($\ddot{N}$,A) = h(R_1($\ddot{N}$, A), R_2($\ddot{N}$, A),−−−−−, R_n($\ddot{N}$, A)) = R_h($\ddot{N}$, A)
⇒ R_h($\ddot{K}$, A)≤ R_h($\ddot{N}$, A)

VII. Application in Decision Making

In this section the newly introduced pythagorean vague binary soft distance measures are used in a decision making problem and the values are compared. An algorithm using in common procedure is given below.

Step 1: Construct PVBSS's ($\ddot{F}$, A) and ($\ddot{G}$, A) based on the given real life situations.
Step 2: Calculate Pythagorean vague binary soft distances between these sets
Step 3: Shortest distance indicates the result

A real life example is given below; Mr. X. wants to purchase a lap-top. Shops under consideration are U_1= {S_1,S_2} for H.P and U_2 = {S_1^*, S_2^*} for DELL. In the selection procedure the parameters under consideration are {e_1= processor, e_2 =RAM and e_3 =Hard drive}. He went to these shops and collected the details which are converted to PVBSS's as follows:

Let the following PVBSS's give data related to qualities wise brands

$$(\ddot{P}_1, A) = \left\{\begin{matrix} \left(e_1, \left(\langle\frac{[0.4,0.6]}{S_1}\rangle, \langle\frac{[0.7,0.1]}{S_2}\rangle, \langle\frac{[0.7,0.8]}{S_1^*}\rangle, \langle\frac{[0.3,0.4]}{S_2^*}\rangle\right)\right) \\ \left(e_2, \left(\langle\frac{[0.5,0.7]}{S_1}\rangle, \langle\frac{[0.6,0.4]}{S_2}\rangle, \langle\frac{[0.8,0.9]}{S_1^*}\rangle, \langle\frac{[0.4,0.3]}{S_2^*}\rangle\right)\right) \\ \left(e_3, \left(\langle\frac{[0.6,0.4]}{S_1}\rangle, \langle\frac{[0.3,0.6]}{S_2}\rangle, \langle\frac{[0.7,0.9]}{S_1^*}\rangle, \langle\frac{[0.8,0.4]}{S_2^*}\rangle\right)\right) \end{matrix}\right\}; \quad (\ddot{P}_2, A) = \left\{\begin{matrix} \left(e_1, \left(\langle\frac{[0.7,0.8]}{S_1}\rangle, \langle\frac{[0.4,0.1]}{S_2}\rangle, \langle\frac{[0.7,0.5]}{S_1^*}\rangle, \langle\frac{[0.6,0.1]}{S_2^*}\rangle\right)\right) \\ \left(e_2, \left(\langle\frac{[0.8,0.1]}{S_1}\rangle, \langle\frac{[0.4,0.7]}{S_2}\rangle, \langle\frac{[0.8,0.9]}{S_1^*}\rangle, \langle\frac{[0.4,0.5]}{S_2^*}\rangle\right)\right) \\ \left(e_3, \left(\langle\frac{[0.6,0.4]}{S_1}\rangle, \langle\frac{[0.3,0.2]}{S_2}\rangle, \langle\frac{[0.7,0.8]}{S_1^*}\rangle, \langle\frac{[0.8,0.6]}{S_2^*}\rangle\right)\right) \end{matrix}\right\},$$

$$(\ddot{P}_3, A) = \left\{\begin{matrix} \left(e_1, \left(\langle\frac{[0.6,0.1]}{S_1}\rangle, \langle\frac{[0.7,0.2]}{S_2}\rangle, \langle\frac{[0.8,0.3]}{S_1^*}\rangle, \langle\frac{[0.9,0.5]}{S_2^*}\rangle\right)\right) \\ \left(e_2, \left(\langle\frac{[0.8,0.1]}{S_1}\rangle, \langle\frac{[0.6,0.5]}{S_2}\rangle, \langle\frac{[0.6,0.9]}{S_1^*}\rangle, \langle\frac{[0.3,0.4]}{S_2^*}\rangle\right)\right) \\ \left(e_3, \left(\langle\frac{[0.6,0.7]}{S_1}\rangle, \langle\frac{[0.8,0.9]}{S_2}\rangle, \langle\frac{[0.3,0.1]}{S_1^*}\rangle, \langle\frac{[0.6,0.7]}{S_2^*}\rangle\right)\right) \end{matrix}\right\}; \quad (\ddot{P}_4, A) = \left\{\begin{matrix} \left(e_1, \left(\langle\frac{[0.5,0.1]}{S_1}\rangle, \langle\frac{[0.2,0.5]}{S_2}\rangle, \langle\frac{[0.6,0.4]}{S_1^*}\rangle, \langle\frac{[0.7,0.9]}{S_2^*}\rangle\right)\right) \\ \left(e_2, \left(\langle\frac{[0.9,0.4]}{S_1}\rangle, \langle\frac{[0.7,0.5]}{S_2}\rangle, \langle\frac{[0.8,0.3]}{S_1^*}\rangle, \langle\frac{[0.8,0.7]}{S_2^*}\rangle\right)\right) \\ \left(e_3, \left(\langle\frac{[0.8,0.7]}{S_1}\rangle, \langle\frac{[0.6,0.4]}{S_2}\rangle, \langle\frac{[0.1,0.6]}{S_1^*}\rangle, \langle\frac{[0.4,0.7]}{S_2^*}\rangle\right)\right) \end{matrix}\right\},$$

Let the following PVBSS's give data related to qualities wise rates

$$(\ddot{Q}_1, A) = \left\{\begin{matrix} \left(e_1, \left(\langle\frac{[0.7,0.8]}{S_1}\rangle, \langle\frac{[0.9,0.2]}{S_2}\rangle, \langle\frac{[0.8,0.9]}{S_1^*}\rangle, \langle\frac{[0.4,0.3]}{S_2^*}\rangle\right)\right) \\ \left(e_2, \left(\langle\frac{[0.4,0.6]}{S_1}\rangle, \langle\frac{[0.5,0.6]}{S_2}\rangle, \langle\frac{[0.9,0.4]}{S_1^*}\rangle, \langle\frac{[0.8,0.1]}{S_2^*}\rangle\right)\right) \\ \left(e_3, \left(\langle\frac{[0.7,0.4]}{S_1}\rangle, \langle\frac{[0.8,0.4]}{S_2}\rangle, \langle\frac{[0.4,0.5]}{S_1^*}\rangle, \langle\frac{[0.6,0.7]}{S_2^*}\rangle\right)\right) \end{matrix}\right\}; \quad (\ddot{Q}_2, A) = \left\{\begin{matrix} \left(e_1, \left(\langle\frac{[0.9,0.1]}{S_1}\rangle, \langle\frac{[0.7,0.2]}{S_2}\rangle, \langle\frac{[0.7,0.4]}{S_1^*}\rangle, \langle\frac{[0.6,0.4]}{S_2^*}\rangle\right)\right) \\ \left(e_2, \left(\langle\frac{[0.7,0.1]}{S_1}\rangle, \langle\frac{[0.5,0.3]}{S_2}\rangle, \langle\frac{[0.7,0.2]}{S_1^*}\rangle, \langle\frac{[0.8,0.3]}{S_2^*}\rangle\right)\right) \\ \left(e_3, \left(\langle\frac{[0.4,0.1]}{S_1}\rangle, \langle\frac{[0.8,0.4]}{S_2}\rangle, \langle\frac{[0.8,0.1]}{S_1^*}\rangle, \langle\frac{[0.4,0.5]}{S_2^*}\rangle\right)\right) \end{matrix}\right\}$$

$$(\ddot{Q}_3, A) = \left\{\begin{matrix} \left(e_1, \left(\langle\frac{[0.4,0.7]}{S_1}\rangle, \langle\frac{[0.5,0.3]}{S_2}\rangle, \langle\frac{[0.9,0.6]}{S_1^*}\rangle, \langle\frac{[0.7,0.4]}{S_2^*}\rangle\right)\right) \\ \left(e_2, \left(\langle\frac{[0.7,0.3]}{S_1}\rangle, \langle\frac{[0.6,0.2]}{S_2}\rangle, \langle\frac{[0.8,0.9]}{S_1^*}\rangle, \langle\frac{[0.7,0.4]}{S_2^*}\rangle\right)\right) \\ \left(e_3, \left(\langle\frac{[0.4,0.5]}{S_1}\rangle, \langle\frac{[0.5,0.2]}{S_2}\rangle, \langle\frac{[0.7,0.3]}{S_1^*}\rangle, \langle\frac{[0.9,0.6]}{S_2^*}\rangle\right)\right) \end{matrix}\right\}; \quad (\ddot{Q}_4, A) = \left\{\begin{matrix} \left(e_1, \left(\langle\frac{[0.6,0.5]}{S_1}\rangle, \langle\frac{[0.8,0.9]}{S_2}\rangle, \langle\frac{[0.7,0.4]}{S_1^*}\rangle, \langle\frac{[0.6,0.1]}{S_2^*}\rangle\right)\right) \\ \left(e_2, \left(\langle\frac{[0.7,0.5]}{S_1}\rangle, \langle\frac{[0.6,0.1]}{S_2}\rangle, \langle\frac{[0.5,0.9]}{S_1^*}\rangle, \langle\frac{[0.6,0.5]}{S_2^*}\rangle\right)\right) \\ \left(e_3, \left(\langle\frac{[0.8,0.6]}{S_1}\rangle, \langle\frac{[0.4,0.9]}{S_2}\rangle, \langle\frac{[0.6,0.7]}{S_1^*}\rangle, \langle\frac{[0.8,0.7]}{S_2^*}\rangle\right)\right) \end{matrix}\right\}$$

Universal sets	S_1	S_2	S_1^*	S_2^*
d_{pvbss}^{nE}	0.47	0.666	0.5882	0.707

Shortest d_{pvbss}^{nE} from calculations indicates qualities wise rates based on brands. d_{pvbss}^{nE} for different sets are given below :
For (1)S_1 = 0.47 (2) S_2 = 0.666 (3) S_1^* = 0.5882 S_2^* = 0.707 . Calculations showed that shortest Normalized Euclidean Pythagorean vague binary soft distance is for show room S$_1$. So better choice for Mr. X is to buy H.P from show-room 1.

VIII. Q-RUNG ORTHOPAIR VAGUE BINARY SOFT SETS

In this section PVBSS's are extended to its higher dimension known as q-rung orthopair VBSS's.

Definition 8.1. (q-rung orthopair vague binary soft sets) Let U_1, U_2 be the common universe and E be a set of parameters with A ⊆ E. Let P(U_1) and P(U_2) denote the power set of q-rung vague subsets on U_1, U_2 respectively. A pair $(\ddot{M}, \mathrm{A})^{\text{q-rung}}$ is said to be a *q-rung orthopair VBSS* denoted as $(\ddot{M}, \mathrm{A})^{\text{q-rung}}$ in U_1, U_2 where $\ddot{M}$ is a mapping given by $\ddot{M}^{\text{q-rung}}$: A→P(U_1)×P(U_2) and $(\ddot{M}, \mathrm{A})^{\text{q-rung}} = \{e_i \in \mathrm{A}/(e_i, \ddot{M}(e_i))\}$where $\ddot{M}(e_i) = \left\{\langle \frac{V^{\text{q-rung}}{}_{\ddot{M}(e_i)}(x_j)}{(x_j)};\ \forall\, e_i \in \mathrm{A}, \forall\, x_j \in U_1 \rangle \langle \frac{V^{\text{q-rung}}{}_{\ddot{M}(e_i)}(y_k)}{(y_k)};\ \forall\, e_i \in \mathrm{A}, \forall\, y_k \in U_2 \rangle\right\}$

$$= \left\{\langle \frac{\left[t^{\text{q-rung}}{}_{\ddot{M}(e_i)}(x_j),\ 1-f^{\text{q-rung}}{}_{\ddot{M}(e_i)}(x_j)\right]}{x_j};\ \forall\, e_i \in \mathrm{A}, \forall\, x_j \in U_1 \rangle, \langle \frac{\left[t^{\text{q-rung}}{}_{\ddot{M}(e_i)}(y_k),\ 1-f^{\text{q-rung}}{}_{\ddot{M}(e_i)}(y_k)\right]}{y_k};\ \forall\, e_i \in \mathrm{A}, \forall\, y_k \in U_2 \rangle\right\}$$

$t^{\text{q-rung}}{}_{\ddot{M}(e_i)}(x_j)$: U_1→ [0, 1] denotes the degree of membership of the element $x_j \in U_1$ in rung stage q ;
$1 - f^{\text{q-rung}}{}_{\ddot{M}(e_i)}(x_j)$: U_1→ [0, 1] denotes the degree of non-membership of the element $x_j \in U_1$ in rung stage q ,
$t^{\text{q-rung}}{}_{\ddot{M}(e_i)}(y_k)$: U_2 → [0,1] denotes the degree of membership of the element $y_k \in U_2$ in the rung stage q;
$1 - f^{\text{q-rung}}{}_{\ddot{M}(e_i)}(y_k)$: U_2 → [0,1] denotes the degree of non-membership of the element $y_k \in U_2$ in rung stage q,
with the condition that,

$$0 \leqslant \left[t^{\text{q-rung}}{}_{\ddot{M}(e_i)}(x_j)\right]^q + \left[1 - f^{\text{q-rung}}{}_{\ddot{M}(e_i)}(x_j)\right]^q \leqslant 1 \text{ and } 0 \leqslant \left[t^{\text{q-rung}}{}_{\ddot{M}(e_i)}(y_k)\right]^q + \left[1 - f^{\text{q-rung}}{}_{\ddot{M}(e_i)}(y_k)\right]^q \leqslant 1$$

IX. CONCLUSIONS

Two universal sets depending on a single parameter set is the highlight of vague binary soft sets. This novel concept enables swiftly moving soft set theory to deal with complex real life situations more flexibly. Pythagorean vague binary soft sets is developed with example. It's stronger form, q-rung orthopair- vague binary soft sets is also developed. Application of Pythagorean concepts on vague binary soft sets will handle application field in a more robust and consistent manner. Validity of basic properties in Classical set theory are also verified for pythagorean vague binary soft sets. It's distance measures are developed with a real life example. Entropy measure with some of it's properties are also given.

REFERENCES

[1] Chang Wang, Anjing Qu, Entropy, similarity measure and distance measure of vague soft sets and their relations, https://dx.doi.org/10.1016/j.ins.2013.05013, Information Sciences 244 (2013), 92-106
[2] Chang Wang, Some properties of entropy of vague soft sets and its applications, journal of intelligent and Fuzzy Systems, DOI: 10.3233/IFS-151608, IOS Press, 29(2015)1443-1452
[3] Duojie Jia-hua, Haidong Zhang and Yanping He, Possibility Pythagorean fuzzy soft set and its application, Journal of Intelligent and Fuzzy Systems, ISSN 1064-1246/19/ 35.00@2019-IOS Press and authors , 36 (2019), 413-421
[4] Dr.Francina shalini.A and Remya.P.B, Vague binary soft sets and their properties, International journal of engineering, science and mathematics, ISSN: 2320-0294, http://www.ijesm.co.in, Vol 7, Issue 11, November (2018), Page 56-73
[5] Harish Garg, A Novel Correlation Coefficients between Pythgorean Fuzzy sets and its Applications to Decision-Making Processes, international Journal of Intelligent Systems, DOI 10.1002/int.21827, Vol.00, 1-19(2016)@2016 Wiley periodicals
[6] Harish Garg, Hesitant Pythagorean fuzzy sets and their aggregation operators in multiple attribute decision-making, International Journal for Uncertainty Quantification, Volume 8, Issue 3, 8(3): (2018), 267-289
[7] Murat Kirisci, New type Pythagorean fuzzy soft set and decision-making application, arXiv:1904.04064v1[math.GM] 5 Apr (2019)
[8] Paul Augustine Ejegwa, Distance and similarity measures for Pythagorean fuzzy sets, Granular computing(2018), https://doi.org/10.1007/s41066-018-00149-z
[9] X.Peng, New operations for interval-valued Pythagorean fuzzy set, doi:10.24200/sci.2018.5142.1119, Scientia Iranica E(2019)26(2), 1049-1076
[10] Ronald R.Yager, Pythagorean Fuzzy Subsets, 978-1-4799-0348-1/13/31.00@2013 IEEE, Electronic ISBN: 978-1-4799-0348-1, DOI: 10.1109/IFSA-NAFIPS.2013.6608375
[11] S.Vinnarasi, F.Nirmala Irudayam, Various Distance Measures on Pythagorean Vague sets, International Journal of Scientific Research in Mathematical and Statistical Sciences, Volume-5, Issue-6,December (2018), pp.244-247
[12] Weibin Deng, Changlin xu, Jin Liu and Feng Hu, A Novel Distance between Vague Sets and Its Applications in Decision Making, Hindawi Publishing Corporation, Mathematical Problems, http://dx.doi.org/10.1155/2014/281095, Article ID 281095,Volume (2014), 10 pages

[13] Xindong Peng and Lin Liu, Information measures for q-rung orthopair fuzzy sets doi:10.1002/int.22115, wileyonlinelibrary.com/journal/int c 2019 Wiley Periodicals, Inc. , Int J Intell Syst. (2019); 34: 1795-1834,
[14] Xindong Peng, New similarity measure and distance measure for Pythagorean fuzzy set, Complex and Intelligent Systems, published online: 07 November(2018), https://doi.org/10.1007/s40747-018-0084-x