Split Domination of Line Segment Graphs

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Abstract:

The split domination number for complementary graphs line segment graph, was introduced by Kulli. Let G be a graph with vertex set V and S be the subset of vertexset V. If every vertex in V-S is adjacent to minimum one vertex of S then S is said to be a dominating set. Let \( \gamma_s(G) \) be the minimum cardinality number of split dominating set. If the induced sub graph \( <V-D> \) is disconnected then the graph contains a split dominating set. In this paper we discussed about split domination for different types of special graphs.

Introduction I:

In 1958, domination was formalized as a theoretical area in graph theory by C. Berge. He referred to the domination number as the co-efficient of external Stability and denoted it \( \beta(G) \). In 1962, Ore was the first to use the term ‘Domination’ number by \( \delta(G) \) and also he introduced the concept of minimal and minimum dominating set of vertices in graph. In 1977, Cockayne and Hedetniemi was introduced the accepted notation \( \gamma(G) \) to denote the domination Number.

The main view of the growth in the number of domination papers is attributed Largely to three factors

1. The diversity of application to real-world and other mathematical
Covering of location problems.
2. Wide variety of domination and its related parameters can be defined.

3. The NP completeness of the basic domination problem, it close and natural relationships to other NP–complete problems and the subsequent interest in finding polynomials time solutions to domination problems in special class of graphs.

The study of split domination number in graphs has found more development in the recent years. The size of smallest dominating set is referred to as the domination number of the graph G has denoted by $\gamma(G)$.

**Definition 2.1: Connected Dominating set**

A dominating Set D is said to be connected dominating set, if the induced subgraph $<D>$ is connected. The connected domination number $\gamma_c(G)$ is the minimum cardinality of a connected dominating set.

**Definition 2.2: Split Dominating set**

If the induced sub graph $<V - D>$ is disconnected, then the dominating set D is called split dominating set.

**Definition 2.3: Split Domination number**

The split dominating number $\gamma_s(G)$ of G is the minimum cardinality of the split dominating set.

**Definition 2.4: Complementary of Crown Graph**

The split domination number of complementary of crown graph is greater than 2.

**Definition 2.5: Minimum split domination set**

A split dominating set D of G is a minimal split Dominating set for each vertex $v \in D$, one of the following conditions holds:

1. There exists a vertex $u \in V-D$ such that $N_G(u) \cap D = \{v\}$.
2. V is an isolated vertex in $<D>$.
3. $<(V-D)\cup\{V\}>$ is connected.

**Definition 2.6:**

Let $G$ be a graph with $V(G) = V_1 \cup V_2 \cup V_3 \cup \ldots \cup V_w \cup U \cup W$ where each $V_i$ is a set having at least two vertices, all having the same $d_2$ and $W = V - UV_i$.

The $d_2$-splitting graph of $G$ denoted by $D2S(G)$ is obtained from $G$ by introducing new vertices $u_1, u_2, \ldots, u_w$ and joining $u_i$ to each vertex of $V_i$ (1 ≤ i ≤ w).
Observation1: For any graph G with n-blocks B(G) ≠ K_p then γ_{sp}(G) ≤ n - 2.

**Theorem 3.1: For any line segment graph** \( l_n \)

\[ γ(l_n) = 1 \] where \( n ≥ 2 \)

**Proof:**

Let \( l_n \) be a line segment graph with n vertices \( V = \{v_1, v_2, v_3 \ldots \ldots v_n \} \). In \( l_n \) graph is a planar undirected graph with \((2n+1)\) vertices and \(3n\) edges. The line segment graph \( l_n \) can be constructed by joining \( n \) copies of the cycle graph \( C_n \) with a common vertex. In \( l_n \) graph one vertices \( v_k \) connected to all the vertices, with the neighborhood \((v_k) = \{v_1', v_2', \ldots \ldots v_n'\} \). Here \((v_k) \cup v_k = V\) and \((v) \cap v_k = v_k\). Therefore the domination number of \( F_n \) graph is 1. i.e. \((l_n) = 1\).

Theorem 2.2

If G is a line segment connected graph with at least one edge, then \( D_2S(G) \) contains a cycle.

**Proof.** Let G be a line segment connected graph with \(|E(G)| ≥ 1\).

Case 1. If G contains a line segment cycle, then \( D_2S(G) \) also contains a cycle.
Case 2. Suppose $G$ contains no line segment cycle. Since $G$ is a connected graph with $|E(G)| \geq 1$
$G$ contains more than one vertex. In a graph with more than one vertex, at least two vertices have the same degree $d_2$. $G$ contains at least two vertices having the same $d_2$
Without loss of generality, let $x$ and $y$ be two vertices in $G$ such that $d_2(x) = d_2(y)$:
By definition of $D_2S(G)$, it contains a vertex $u$ such that $u$ is adjacent with both $x$ and $y$:
Subcase 1. If $x$ and $y$ are not adjacent, then they are connected by a path
$x = v_1, v_2, ..., v_n = y$. Since $G$ is connected, $u, v_1, ..., v_n, u$ is a cycle in $D_2S(G)$.
Subcase 2 If $x$ and $y$ are adjacent, then $u, x, y, u$ form a cycle in $D_2S(G)$.

2. Conclusion

In this paper, we have discussed on the split domination of line segment domination graph. In future we propose to extend this work with many graphs.

Reference


