A NEW STOCHASTIC MODEL ON THE GENERALIZATION OF SUJATHA DISTRIBUTION FOR THE EFFECT OF TWO TYPES OF EXERCISE ON PLASMA GROWTH HORMONE

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Abstract:

The Generalization of Sujatha distribution is a very famous stochastic mathematical model through its real life applications. A Generalization of Sujatha distribution is one of the lifetime or stochastic model with some statistical properties. In this study, we have selected the real time medical data i.e., the effect of two types of exercise on plasma growth hormone in equal duration. The results clearly states that the levels of probability density function and hazard function of two types of exercise on plasma growth hormone. However, it has been recorded that the application part is well fitted with the stochastic model and this could be an accepted stochastic model to express the life time data in a well identified manner.

Key words: Sujatha distribution, Hazard function, Exercise, Growth hormone.

1. INTRODUCTION

The stochastic models of lifetime data are essential in all applied sciences including engineering, medical science, financial and many other fields. Several lifetime distributions have expanded in statistical literature, including exponential, Poisson, Lindley, Akash, Shanker, gamma, Gumble and Weibull distributions. Each distribution has of its own improvement and disadvantages in mathematical modelling of real-time data. In past years, many work have been done on Lindley distribution, its mixture with other distributions, extensions, and generalizations. The probability distribution function and probability distribution function of Sujatha distribution introduced by Shanker et al. 2016 [1] has been developed with some important. There, its important properties including hazard rate function, mean residual life function, stochastic ordering, mean deviations, Bonferroni and Lorenz curves, stress-strength reliability have also been discussed. Further, Shanker et al. 2017 [2] introduced a generalization of Sujatha distribution in 2017.

Since generalization of Sujatha distribution (AGSD) includes Lindley and Sujatha distributions as some familiar cases, it is expected to give better fit than both Lindley and Sujatha distributions for modelling lifetime data from biological sciences and engineering.

Growth hormone is a peptide hormone that stimulates growth, cell reproduction, and cell regeneration in humans and other animals [10]. It is thus important in human development and it affects bone density, lipid metabolism and muscle growth in adults. Kindermann et al. 1982 [5], have demonstrated that the variable and inconsistent effects of growth hormone for the different types of exercise in the human body. Kaliraja et al. 2018 [7], have demonstrated the the effect of acute exercise on serum growth hormone (GH) levels in elite water polo players by using Weibull distribution. In this paper, we applied the generalization of Sujatha distribution with its application to fit in to the published work of [6] that the effect of two types of exercise of equal duration and work expenditure on plasma growth hormone levels.

2. METHODOLOGY

2.1. MATHEMATICAL MODEL

2.1.1. GENERALIZATION OF SUJATHA DISTRIBUTION

The generalization of Sujatha distribution is a fantastic tool of mathematical modelling, its application used in various areas of real life problems and Rama sankar recently developed of its properties and fundamental work. Hence Rama sankar et al. 2016[3], improved probability density function and cumulative distribution function of Aradhana distribution are given as follows,

$$f(x,\theta) = \frac{\theta^3}{\theta^2 + 2\theta + 2}\,(1+x)^2 e^{-\theta x} \quad ,x > 0, \theta > 0.$$

$$F(x,\theta) = 1 - \left[1 + \frac{\theta x + (\theta x + 2\theta + 2)}{\theta^2 + 2\theta + 2}\right] e^{-\theta x} \quad ,x > 0, \theta > 0.$$

Rama sanker et al. 2016 [4] developed Sujatha distribution after derived some properties. The probability density function and cumulative distribution function of Sujatha distribution written of the form is given below,

$$f(x,\theta) = \frac{\theta^3}{\theta^2 + 2\theta + 2}\,(1+x+x^2) e^{-\theta x} \quad ,x > 0, \theta > 0.$$

$$F(x,\theta) = 1 - \left[1 + \frac{\theta x + (\theta x + \theta + 2)}{\theta^2 + \theta + 2}\right] e^{-\theta x} \quad ,x > 0, \theta > 0.$$

The generalization of Sujatha distribution developed by Rama sankar et al. 2017[2].The probability density function is written of the form as follows,

$$f(x,\theta,\alpha) = \frac{\theta^3}{\theta^2+\theta+2\alpha}\,(1+x+\alpha x^2)e^{-\theta x} \quad ,x>0,\theta>0,\alpha>0.$$

Where the parameters are selected values i.e. positive values are possible but greater than zero. The cumulative distribution function defined as,

$$F(x,\theta,\alpha) = 1-\left[1+\frac{\theta x(\alpha\theta x+\theta+2\alpha)}{\theta^2+\theta+2\alpha}\right] e^{-\theta x} \quad ,x>0,\theta>0.$$

Moments of generalization of Sujatha distribution derivation as follows,

The r^{th} moment about origin μ_1' of a generalization of Sujatha distribution obtained from above equation,

$$\mu_r' = \frac{r!\,[\theta^2+(r+1)\theta+(r+1)(r+2)\alpha]}{\theta^r(\theta^2+\theta+2\alpha)} \;,r=1,2,3\ldots..$$

Let X be a continuous random variable with $f(x)$ and$F(x)$. The hazard rate function is failure rate function which is known us, of its hazard rate function of X can defined as,

$$h(x) = \lim_{\Delta x\to 0}\frac{P(X<x+\Delta x|X>x)}{\Delta x} = \frac{f(x)}{1-F(x)}$$

Corresponding hazard rate function obtained from above equation then we get,

$$h(x) = \frac{\theta^3(1+x+\alpha x^2)}{\theta x(\alpha\theta x+\theta+2\alpha)+(\theta^2+\theta+2\alpha)}$$

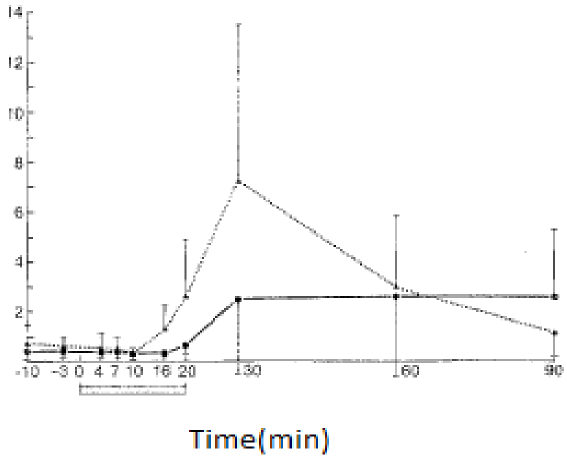
3. RESULTS

3.1. APPLICATION

3.1.1. Background

To inaugurate a generalization of Sujatha distribution, the life time data was adopted from the published work of [6], they have performed the study with participated five normal men in an exercise program twice weekly volunteered as subjects. Two types of exercise were compared there are aerobic (I) and anaerobic (II). All subjects participated seven times of three weeks in both exercise the subjects pedalled according to the given pace and the revolutions of the flywheel were counted simultaneously.

The total external work performed in both exercise nearly equal (120.0 kJ vs. 119.7 kJ in I and II, respectively) as was the total duration (20 min). Every session, subsequent antecubital venous blood tests were taken by means of an inhabiting catheter at the times$-10, -3, 4, 7, 10, 16, 20, 30, 60,$ and 90 min concerning the start of activity (0 time). GH levels in the plasma were analysed.

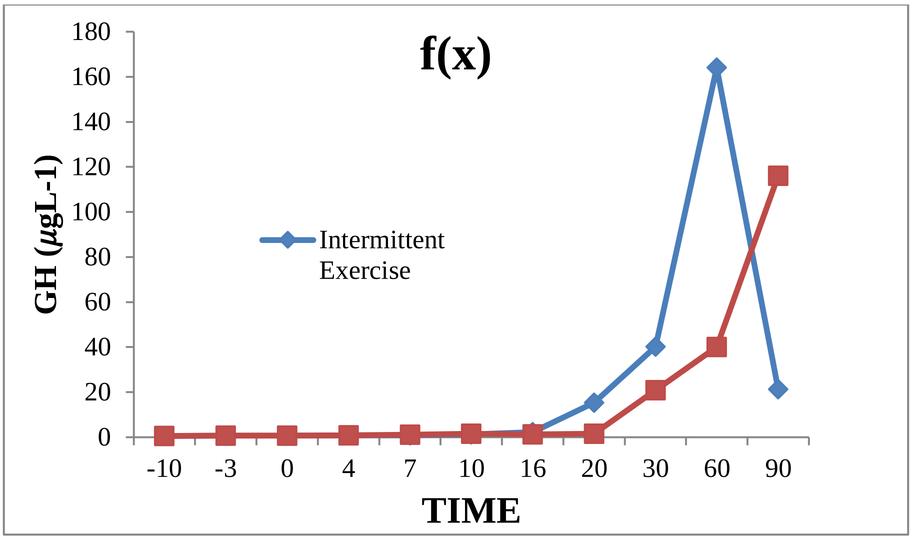


Medical Figure: 1. Plasma growth hormone levels before during, and after continuous and intermittent cycling in five normal males.

4. MATHEMATICAL RESULTS

4.1. The probability density function of the generalization of Sujatha distribution

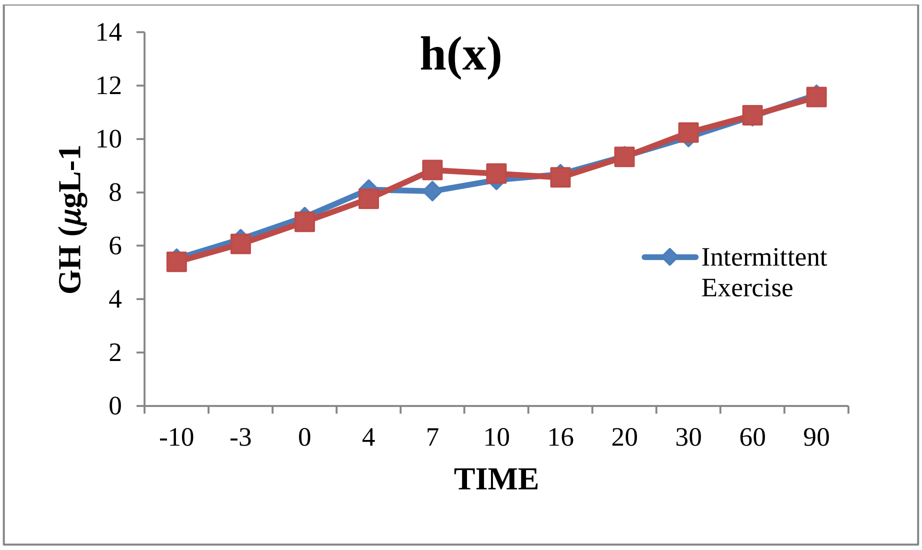
Probability density functions $f(x)$ of generalization of Sujatha distribution analysis on the Plasma growth hormone levels before, during, and after continuous and intermittent cycling in five normal males of both exercises (aerobic (I) and anaerobic (II)) are shown in mathematical figure 1. Generalization of Sujatha distribution probability density function plots reveals that the evaluated level of GH in intermittent exercises as compared with continuous exercises for the corresponding time intervals of $-10, -3, 4, 7, 10, 16, 20, 30, 60,$ and 90 min.



Mathematical Figure: 1. Graph of probability density function of Sujatha distribution for selected values of parameters

4.2. The Hazard function of the generalization of Sujatha distribution

Mathematical Figure. 2 represents the hazard rate function $h(x)$ of generalization of Sujatha distribution plot of Plasma growth hormone levels in before, during, and after continuous and intermittent cycling in five normal males of both exercises (aerobic (I) and anaerobic (II)) at the times intervals of $-10, -3, 4, 7, 10, 16, 20, 30, 60$, and 90 min. The results in hazard rate functions plots are shows that there is no much changes in the values of GH levels in intermittent exercise and continuous exercise.



Mathematical Figure: 1. Graph of hazard function of Sujatha distribution for selected values of parameters

5. DISCUSSION:

Statistical distribution models are especially useful in recognizing and predicting the real time global phenomena. One among them is the extreme value distribution. It has been expansively used to model lifetime data and modelling natural phenomena. In this paper we have evaluated the GH levels in two types of exercise aerobic (I) and anaerobic (II) using generalization of Sujatha distribution. Here, we have use d the selected value parameters $\alpha > 0$ and $\theta > 0$,. The obtained results are reveals that the growth hormone levels in the probability density function $f(x)$ were significantly increases (shown in mathematical figure 1) in intermittent exercise as compared with the continuous exercise in five males subjects at the times,$-10, -3, 4, 7, 10, 16, 20, 30, 60$, and 90 min. However, it should be noted that there is no much difference in the values of the hazard rate function $h(x)$ of intermittent exercise and continuous exercise levels of growth hormone intermittent exercise and continuous exercise of both types of exercise (shown in mathematical figure 2). The results of the present study are matches with our previously published work (Kaliraja et al. [8 &9]).

6. CONCLUSON:

In conclusion, we have elucidated the mathematical interpretation using the generalization of Sujatha distribution with its fundamental stochastic properties. The obtained results expose that the probability density function $f(x)$ and hazard function $h(x)$ plots for the levels of growth hormone in intermittent exercise and continuous exercise of five male subjects at $-10, -3, 4, 7, 10, 16, 20, 30, 60$ and 90 min, respectively. Our results are well coincides with the life time data of Vanhelder et al., suggesting that this could be an accepted stochastic model.

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