Vertex Antimagic Edge Labeling Of Tessellation Of K_4

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Abstract - **The concept of grid graphs and extended grid graphs have been one of the much sought after topics in graph theory as they have rich and wise applications. Graphs like extended grid, Mongolian tent etc. have been used by researchers to prove a variety of properties. One such extended grid graph is the Tessellation of K_4. In this paper, we apply the concept of antimagic labeling to the graph Tessellation of K_4.**

Keywords - **Complete Graph, Weight, Labeling, Tessellation, Vertex Antimagic Edge Labeling**
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1. INTRODUCTION

The concept of labeling the vertices and edges has come a long way with researchers introducing a spate of labeling and one such is the magic and antimagic labeling. The zeal behind the development of this paper is the wide range of applications for the grid graphs.

2. VERTEX ANTIMAGIC EDGE LABELING

In graph theoretic parlance, the weight of a vertex or an edge is the label assigned to that vertex or the edge. The labeling has been one of the intriguing topics in graph theory which has been effectively used by researchers. A connected graph $(V(G),E(G))$, as defined by Baca and Miller [4], is said to be an (a, d) antimagic edge labelled graph if there exists a positive integer a and a non-negative integer d and a bijection $f_1:E\rightarrow\{1,2,\ldots,|E(G)|\}$ such that the induced mapping g: $V(G)\rightarrow W$, where $W = \{a,a+d,a+2d,\ldots,a+|V-1|d\}$is also a bijection. J. Sedlacek [8] was the first to introduce the concept of magic labeling. It was J.A. Macdougall, M. Miller and W.D. Wallis [5] [6] who extended it to vertex magic labeling. One of the pioneers in labelling, N. Hartsfield and J. Ringel [3] introduced the notion of antimagic labeling. But it was R. Bodendiek and G. Walther [1] who introduced the concept of (a,d) antimagic labeling. The concept of antimagic labeling was handled extensively by J.A. Gallian [2].

3. TESSELLATION OF K_4

Tessellation as defined by Richard Xu and Sergiy Merenkov [7] is a collection of geometric shapes with no overlaps and non-empty space between them. A connected graph is said to be complete if there exist an edge between every pair of distinct vertices. Tessellation of K_4 is an extended grid of K_4. We denote the tessellation of K_4 by $T(m,n)$ where n is the number of columns and m is the number of rows. The number of vertices in $T(m,n)$ is $mn + m + n$ and edges is $4mn + m + n$. In this paper, we show that the graph $T(m,n)$ admits vertex antimagic edge labeling.

4. MAIN RESULTS

Theorem 1: The tessellation of K_4 $T(1,n)$, $n \geq 3$ admits vertex antimagic edge labeling.

Proof: To prove that the graph $T(1,n)$, $n \geq 3$ admits vertex antimagic edge labeling, we label the edges of the graph by defining a function $f_1:E\rightarrow R$ as follows

$f_1(u_{1j}u_{1(j+1)}) = j+1, 1 \leq j \leq n$

$f_1(u_{ij}u_{(i+1)j}) = n+2, i = 1, j = n+1$

$f_1(u_{(i+1)j}u_{(i+1)(j+1)}) = m+2n+2-j, i = 1, 1 \leq j \leq n$

$f_1(u_{ij}u_{(i+1)j}) = j, i = j = 1$

$f_1(u_{ij}u_{(i+1)(j+1)}) = 2(m+n)+3j-2, i = 1, 1 \leq j \leq n$

$f_1(u_{i(j+1)}u_{(i+1)j}) = 2(m+n)+3j-1, i = 1, 1 \leq j \leq n$

$f_1(u_{ij}u_{(i+1)j}) = 2(m+n)+3(j-1), i = 1, 2 \leq j \leq n$

Claim: We claim that the edge labels are all distinct.

Case – 1: For $j \neq k$ in $1 \leq j \leq n$, assume that $f_1(u_{1j}u_{1(j+1)}) = f_1(u_{1k}u_{1(k+1)})$

$\Rightarrow j+1 = k+1 \Rightarrow j = k$, a contradiction

$\Rightarrow$the edge labels are all distinct

Case – 2: For $i = 1$and $j = n+1$, the edge label defined by $f_1(u_{ij}u_{(i+1)j}) = n+2$ is fixed.

Case – 3: For $i = 1$ and $j \neq k$ in $1 \leq j \leq n$, assume that $f_1(u_{(i+1)j}u_{(i+1)(j+1)}) = f_1(u_{(i+1)k}u_{(i+1)(k+1)})$

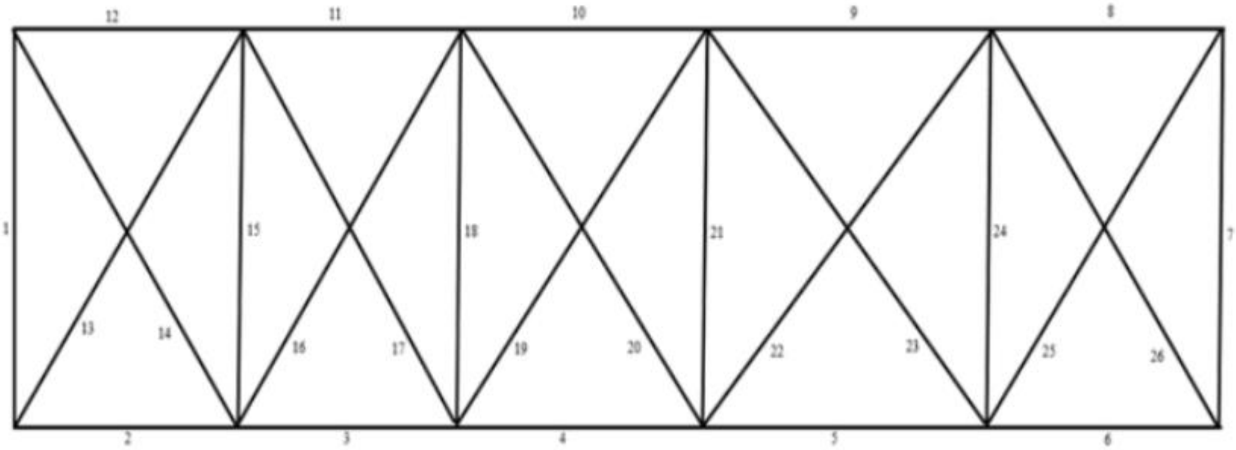
$\Rightarrow m+2n+2-j = m+2n+2-k \Rightarrow j = k$, a contradiction

$\Rightarrow$the edge labels are all distinct.

Case – 4: For $i = j = 1$, the edge label defined by $f_1(u_{ij}u_{(i+1)j}) = j$ is fixed.
Case – 5: For $j \neq k$ in $1 \leq j \leq n$ and $i = 1$, assume that $f_1(u_{ij}u_{(i+1)(j+1)}) = f_1(u_{ik}u_{(i+1)(k+1)})$
$\Rightarrow 2(m + n)+3j - 2 = 2(m + n)+3k - 2 \Rightarrow j = k$, a contradiction
$\Rightarrow$the edge labels are all distinct.
Case – 6: For $j \neq k$ in $1 \leq j \leq n$ and $i = 1$, assume that $f_1(u_{i(j+1)}u_{(i+1)j}) = f_1(u_{i(k+1)}u_{(i+1)k})$
$\Rightarrow 2(m + n)+3j - 1 = 2(m + n)+3k - 1 \Rightarrow j = k$, a contradiction
$\Rightarrow$the edge labels are all distinct.
Case – 7: For $j \neq k$ in $2 \leq j \leq n$ and $i = 1$, assume that $f_1(u_{ij}u_{(i+1)j}) = f_1(u_{ik}u_{(i+1)k})$
$\Rightarrow 2(m + n)+3(j - 1) = 2(m + n)+3(k - 1) \Rightarrow j = k$, a contradiction
$\Rightarrow$the edge labels are all distinct.
Case – 8: For $i = 1$, j_1 in $1 \leq j \leq n$, and $j_2 = n + 1$, assume that $f_1(u_{1j}u_{1(j+1)}) = f_1(u_{ij}u_{(i+1)j})$
$\Rightarrow j_1 + 1 = n+ 2 \Rightarrow j_1 = n + 1$, a contradiction
$\Rightarrow$the edge labels are all distinct
Case – 9: For $i = 1$and j in $1\leq j \leq n$, assume that $f_1(u_{1j}u_{1(j+1)}) = f_1(u_{(i+1)j}u_{(i+1)(j+1)})$
$\Rightarrow j + 1 = m + 2n + 2 - j \Rightarrow 2j = m + 2n + 1$, a contradiction
$\Rightarrow$the edge labels are all distinct
Case – 10: For $i = 1$, j_1 in $1\leq j \leq n$ and $j_2 = 1$, assume that $f_1(u_{1j_1}u_{1(j_1+1)}) = f_1(u_{mj_2}u_{(m+1)j_2})$
$\Rightarrow j_1 + 1 = j_2 \Rightarrow j_1 - j_2 =- 1$, a contradiction
$\Rightarrow$the edge labels are all distinct.
Case – 11: For j in $1 \leq j \leq n$ and for $i= 1$, assume that $f_1(u_{1j}u_{1(j+1)}) = f_1(u_{ij}u_{(i+1)(j+1)})$
$\Rightarrow j + 1 = 2(m + n)+3j - 2 \Rightarrow 3 = 2(m + n)+2j$, a contradiction
$\Rightarrow$the edge labels are all distinct.
Case – 12: For j in $1\leq j \leq n$ and for $i = 1$, assume that $f_1(u_{1j}u_{1(j+1)}) = f_1(u_{i(j+1)}u_{(i+1)j})$
$\Rightarrow j + 1 = 2(m + n)+3j - 1 \Rightarrow 1 = m + n+j$, a contradiction
$\Rightarrow$the edge labels are all distinct.
Case – 13: For $i = 1$, j_1 in $1 \leq j \leq n$ and j_2 in $2 \leq j \leq n$, assume that $f_1(u_{1j_1}u_{1(j_1+1)}) = f_1(u_{ij_2}u_{(i+1)j_2})$
$\Rightarrow j_1 + 1 = 2(m + n)+3j_2 - 3 \Rightarrow 4 = 2(m + n)+ 3j_2 - j_1$, a contradiction
$\Rightarrow$the edge labels are all distinct.
Case – 14: For $i= 1$ and j in $1\leq j \leq n$, assume that $f_1(u_{ij}u_{(i+1)j}) = f_1(u_{(i+1)j}u_{(i+1)(j+1)})$
$\Rightarrow n + 2 = m + 2n + 2 - j \Rightarrow j = m + n$, a contradiction
$\Rightarrow$the edge labels are all distinct.
Case – 15: For $i= 1$, j_1 in $1 \leq j \leq n$ and $j_2 = 1$, assume that $f_1(u_{ij_1}u_{(i+1)j_1}) = f_1(u_{ij_2}u_{(i+1)j_2})$
$\Rightarrow n + 2 = j_2 \Rightarrow n = -1$, a contradiction
$\Rightarrow$the edge labels are all distinct.
Case – 16: For $i= 1$ and j in $1\leq j \leq n$, assume that $f_1(u_{ij}u_{(i+1)j}) = f_1(u_{ij}u_{(i+1)(j+1)})$
$\Rightarrow n + 2 = 2(m+n) + 3j - 2 \Rightarrow 4 = 2m + n + 3j$, a contradiction
$\Rightarrow$the edge labels are all distinct.
Case – 17: For j in $1\leq j \leq n$ and for $i = 1$, assume that $f_1(u_{ij}u_{(i+1)j}) = f_1(u_{i(j+1)}u_{(i+1)j})$
$\Rightarrow n + 2 = 2(m + n)+3j - 1 \Rightarrow 3 = 2m + n + 3j$, a contradiction
$\Rightarrow$the edge labels are all distinct.
Case – 18: For $i = 1$, j_1 in $1 \leq j \leq n$ and j_2 in $2 \leq j \leq n$, assume that $f_1(u_{ij_1}u_{(i+1)j_1}) = f_1(u_{ij_2}u_{(i+1)j_2})$
$\Rightarrow n + 2 = 2(m + n)+3j_2 - 3 \Rightarrow 5 = 2m + n +3 j_2$, a contradiction
$\Rightarrow$the edge labels are all distinct.
Case – 19: For $i = 1$, j_1 in $1 \leq j \leq n$ and $j_2 = 1$, assume that $f_1(u_{(i+1)j_1}u_{(i+1)(j_1+1)}) = f_1(u_{ij_2}u_{(i+1)j_2})$
$\Rightarrow m + 2n + 2 - j_1 = j_2 \Rightarrow m + 2n + 1 = j_1$, a contradiction
$\Rightarrow$the edge labels are all distinct
Case – 20: For $i = 1$and j in $1\leq j \leq n$, assume that $f_1(u_{(i+1)j}u_{(i+1)(j+1)}) = f_1(u_{ij}u_{(i+1)(j+1)})$
$\Rightarrow m + 2n + 2 - j = 2(m + n) + 3j - 2 \Rightarrow m + 4j = 4$, a contradiction
$\Rightarrow$the edge labels are all distinct.
Case – 21: For j in $1\leq j \leq n$ and $i=1$, assume that $f_1(u_{(i+1)j}u_{(i+1)(j+1)}) = f_1(u_{i(j+1)}u_{(i+1)j})$
$\Rightarrow m + 2n + 2 - j = 2(m + n) + 3j - 1 \Rightarrow 3 = m + 4j$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 22: For $i = 1$, j_1 in $1 \le j \le n$ and j_2 in $2 \le j \le n$, assume that $f_1(u_{(i+1)j_1}u_{(i+1)(j_1+1)}) = f_1(u_{ij_2}u_{(i+1)j_2})$
$\Rightarrow m + 2n + 2 - j_1 = 2(m + n) + 3j_2 - 3 \Rightarrow 5 = m + j_1 + 3j_2$, a contradiction
$\Rightarrow$the edge labels are all distinct.
Case – 23: For $i = j_1 = 1$ and j_2 in $1 \le j \le n$, assume that $f_1(u_{i1}u_{(i+1)j_1}) = f_1(u_{ij_2}u_{(i+1)(j_2+1)})$
$\Rightarrow j_1 = 2(m + n) + 3j_2 - 2 \Rightarrow 3 = 2(m + n) + 3j_2$, a contradiction
$\Rightarrow$the edge labels are all distinct.
Case – 24: For $i = j_1 = 1$ and j_2 in $1 \le j \le n$, assume that $f_1(u_{ij_1}u_{(i+1)j_1}) = f_1(u_{i(j_2+1)}u_{(i+1)j_2})$
$\Rightarrow j_1 = 2(m + n) + 3j_2 - 1 \Rightarrow 2 = 2(m + n) + 3j_2$, a contradiction
$\Rightarrow$the edge labels are all distinct.
Case – 25: For $i = j_1 = 1$ and j_2 in $2 \le j \le n$, assume that $f_1(u_{ij_1}u_{(i+1)j_1}) = f_1(u_{ij_2}u_{(i+1)j_2})$
$\Rightarrow j_1 = 2(m + n) + 3j_2 - 3 \Rightarrow 1 = 2(m + n) + 3j_2 - 3 \Rightarrow 4 = 2(m + n) + 3j_2$, a contradiction
$\Rightarrow$the edge labels are all distinct.
Case – 26: For $i = 1$ and j in $1 \le j \le n$, assume that $f_1(u_{ij}u_{(i+1)(j+1)}) = f_1(u_{i(j+1)}u_{(i+1)j})$
$\Rightarrow 2(m + n) + 3j - 2 = 2(m + n) + 3j - 1 \Rightarrow 2 = 1$, absurd
$\Rightarrow$the edge labels are all distinct.
Case – 27: For $i = 1$, j_1 in $1 \le j \le n$ and $j_2 = 2 \le j \le n$, assume that $f_1(u_{ij_1}u_{(i+1)(j_1+1)}) = f_1(u_{ij_2}u_{(i+1)j_2})$
$\Rightarrow 2(m + n) + 3j_1 - 2 = 2(m + n) + 3j_2 - 3 \Rightarrow j_2 - j_1 = 1/3$, a contradiction
$\Rightarrow$the edge labels are all distinct.
Case – 28: For $i = 1$, j_1 in $1 \le j \le n$ and j_2 in $2 \le j \le n$, assume that $f_1(u_{i(j_1+1)}u_{(i+1)j_1}) = f_1(u_{ij_2}u_{(i+1)j_2})$
$\Rightarrow 2(m + n) + 3j_1 - 1 = 2(m + n) + 3j_2 - 3 \Rightarrow j_2 - j_1 = 2/3$, a contradiction
$\Rightarrow$the edge labels are all distinct.

From the cases discussed above, it follows that the edge labels are all distinct. The label of a vertex is the sum of labels of the edges that are incident with it. Since the edge labels are all distinct, it follows that the vertex labels are all distinct. Hence the graph $T(1,n)$, $n \ge 3$ admits vertex antimagic edge labeling. As an illustration, we consider the graph of $T(1,5)$



Tessellation of K_4 T(1,5)

Theorem – 2: The tessellation of K_4 denoted by $T(2,n)$, $n \ge 3$ admits vertex antimagic edge labeling.
Proof: To prove that the graph $T(2,n)$, $n \ge 3$ admit vertex antimagic edge labeling, we label the edges of the graph by defining a function $f_1:E \to R$ as follows;

$f_1(u_{ij}u_{i(j+1)}) = i + j - 1,$ $\quad i = 3, \quad 1 \le j \le n$
$f_1(u_{(i+1)j}u_{ij}) = j - i + 4,$ $\quad i = 1,2 \quad j = n + 1$
$f_1(u_{1j}u_{1(j+1)}) = 2(m + n) + 1 - j,$ $\quad 1 \le j \le n$
$f_1(u_{i1}u_{(i+1)1}) = i,$ $\quad i = 1,2$
$f_1(u_{(i+1)j}u_{i(j+1)}) = 2(m + n + j) - 1,$ $\quad i = 1 \quad 1 \le j \le n$
$f_1(u_{(i+1)(j+1)}u_{ij}) = 2(m + n + j),$ $\quad i = 1 \quad 1 \le j \le n$
$f_1(u_{(i+1)j}u_{i(j+1)}) = 2(m + 2n + j) - 1,$ $\quad i = 2 \quad 1 \le j \le n$
$f_1(u_{(i+1)(j+1)}u_{ij}) = 2(m + 2n + j),$ $\quad i = 2 \quad 1 \le j \le n$
$f_1(u_{ij}u_{i(j+1)}) = 4mn + i + j,$ $\quad i = 2 \quad 1 \le j \le n$
$f_1(u_{ij}u_{(i+1)j}) = 2mn + 2m + 3n + 1 - j,$ $\quad i = 1 \quad 2 \le j \le n$
$f_1(u_{ij}u_{(i+1)j}) = 4(mn + 1) - j,$ $\quad i = 2 \quad 2 \le j \le n$

Claim: We claim that the edge labels are all distinct.

Case – 1: For $j \neq k$ in $1 \leq j \leq n$ and $i = 3$, assume that $f_1(u_{ij}u_{i(j+1)}) = f_1(u_{ik}u_{i(k+1)})$
$\Rightarrow i + j - 1 = i + k - 1 \Rightarrow j = k$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 2: For $j = n + 1$ and $i = 1, 2$ the edge label defined by $f_1(u_{(i+1)j}u_{ij}) = j - i + 4$ is fixed.

Case – 3: For $j \neq k$ in $1 \leq j \leq n$, assume that $f_1(u_{1j}u_{1(j+1)}) = f_1(u_{1k}u_{1(k+1)})$
$\Rightarrow 2(m + n) + 1 - j = 2(m + n) + 1 - k \Rightarrow j = k$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 4: For $i = 1, 2$ the edge label defined by $f_1(u_{i1}u_{(i+1)1})= i$ is fixed.

Case – 5: For $j \neq k$ in $1 \leq j \leq n$ and $i = 1$, assume that $f_1(u_{(i+1)j}u_{i(j+1)}) = f_1(u_{(i+1)k}u_{i(k+1)})$
$\Rightarrow 2(m + n + j) - 1 = 2(m + n + k) - 1 \Rightarrow j = k$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 6: For $j \neq k$ in $1 \leq j \leq n$ and $i = 1$, assume that $f_1(u_{(i+1)(j+1)}u_{ij}) = f_1(u_{(i+1)(k+1)}u_{ik})$
$\Rightarrow 2(m + n + j) = 2(m + n + k) \Rightarrow j = k$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 7: For $j \neq k$ in $1 \leq j \leq n$ and for and $i = 2$,assume that $f_1(u_{(i+1)j}u_{i(j+1)}) = f_1(u_{(i+1)k}u_{i(k+1)})$
$\Rightarrow 2(m + 2n + j) - 1 = 2(m + 2n + k) - 1, \Rightarrow j = k$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 8: For $j \neq k$ in $1 \leq j \leq n$ and for $i = 2$, assume that $f_1(u_{(i+1)(j+1)}u_{ji}) = f_1(u_{(i+1)(k+1)}u_{ki})$
$\Rightarrow 2(m + 2n + j) = 2(m + 2n + k) \Rightarrow j = k$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 9: For $j \neq k$ in $1 \leq j \leq n$ and for $i = 2$, assume that $f_1(u_{ij}u_{i(j+1)}) = f_1(u_{ik}u_{i(k+1)})$
$\Rightarrow 4mn + i + j = 4mn + i + k \Rightarrow j = k$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 10: For $j \neq k$ in $2 \leq j \leq n$ and for $i = 1$, assume that $f_1(u_{ij}u_{(i+1)j}) = f_1(u_{ik}u_{(i+1)k})$
$\Rightarrow 2mn + 2m + 3n + 1 + j = 2mn + 2m + 3n + 1 + k \Rightarrow j = k$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 11: For $j \neq k$ in $2 \leq j \leq n$ and for $i = 2$, assume that $f_1(u_{ij}u_{(i+1)j}) = f_1(u_{ij}u_{(i+1)j})$
$\Rightarrow 4(mn + 1) - j = 4(mn + 1) - k \Rightarrow j = k$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 12: For j_1 in $1 \leq j \leq n$, $i_1 = 3$, $i_2 = 1,2$ and $j_2 = n + 1$, assume that $f_1(u_{i_1j_1}u_{i_1(j_1+1)}) = f_1(u_{(i_2+1)j_2}u_{i_2j_2})$,
$\Rightarrow i_1 + j_1 - 1 = j_2 - i_2 + 4 \Rightarrow i_2 + j_1 - j_2 = 2$, a contradiction
$\Rightarrow$the edge labels are all distinct

Case – 13: For j in $1 \leq j \leq n$ and for $i = 3$, assume that $f_1(u_{ij}u_{i(j+1)}) = f_1(u_{1j}u_{1(j+1)})$
$\Rightarrow i + j - 1 = 2(m + n) + 1 - j \Rightarrow 2j = 2(m + n) - 1$, a contradiction since the LHS is even and RHS is odd
$\Rightarrow$the edge labels are all distinct.

Case – 14: For $i_1 = 3$, $i_2 = 1, 2$ and j in $1 \leq j \leq n$, assume that $f_1(u_{i_1j}u_{i_1(j+1)}) = f_1(u_{i_21}u_{(i_2+1)1})$
$\Rightarrow i_1 + j - 1 = i_2 \Rightarrow i_1 - i_2 + j = 1$, a contradiction
$\Rightarrow$the edge labels are all distinct

Case – 15: For $i_1 = 3$, $i_1 = 1$ and j in $1 \leq j \leq n$, assume that $f_1(u_{ij}u_{i(j+1)}) = f_1(u_{(i+1)j}u_{i(j+1)})$
$\Rightarrow i_1 + j - 1 = 2(m + n + j) - 1 \Rightarrow 3 = 2(m + n) + j$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 16: For $i_1 = 3$, $i_2 = 1$ and j in $1 \leq j \leq n$, assume that $f_1(u_{i_1j}u_{i_1(j+1)}) = f_1(u_{(i_2+1)(j+1)}u_{i_2j})$
$\Rightarrow i_1 + j - 1 = 2(m + n + j) \Rightarrow 2 = 2(m + n) + j$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 17: For $i_1 = 3$, $i_2 = 2$ and j in $1 \leq j \leq n$, assume that $f_1(u_{i_1j}u_{i_1(j+1)}) = f_1(u_{(i_2+1)j}u_{i_2(j+1)})$
$\Rightarrow i_1 + j - 1 = 2(m + 2n + j) - 1 \Rightarrow 3 = 2(m + 2n) + j$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 18: For $i_1 = 3$, $i_2 = 2$ and j in $1 \leq j \leq n$, assume that $f_1(u_{i_1j}u_{i_1(j+1)}) = f_1(u_{(i_2+1)(j+1)}u_{i_2j})$
$\Rightarrow i_1 + j - 1 = 2(m + 2n + j) \Rightarrow -j = 2(m + 2n - 1)$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 19: For $i_1 = 3$, $i_2 = 2$ and j in $1 \leq j \leq n$, assume that $f_1(u_{i_1j}u_{i_1(j+1)}) = f_1(u_{i_2j}u_{i_2(j+1)})$
$\Rightarrow i_1 + j - 1 = 4mn + i_2 + j \Rightarrow mn = 0$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 20: For $i_1 = 3$, $i_2 = 1$, j_1 in $1 \leq j \leq n$ and j_2 in $2 \leq j \leq n$, assume that

$f_1(u_{i_1 j_1} u_{i_1 (j_1+1)}) = f_1(u_{i_2 j_2} u_{(i_2+1) j_2}) \Rightarrow i_1 + j_1 - 1 = 2(m + n + mn) + n - j_2 + 1$
$\Rightarrow j_1 + j_2 = 2m + 3n + 2mn - 1$, a contradiction.
$\Rightarrow$the edge labels are all distinct.

Case – 21: For $i_1 = 3$, $i_2 = 2$, j_1 in $1 \le j \le n$ and j_2 in $2 \le j \le n$, assume that
$f_1(u_{i_1 j_1} u_{i_1 (j_1+1)}) = f_1(u_{i_2 j_2} u_{(i_2+1) j_2})$. $\Rightarrow i_1 + j_1 - 1 = 4(mn + 1) - j_2 \Rightarrow j_1 + j_2 = 4mn + 2$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 22: For $i = 1, 2, j_1 = n + 1$ and $j_2 = 1 \le j \le n$, assume that $f_1(u_{(i+1) j_1} u_{i j_1}) = f_1(u_{1 j_2} u_{1 (j_2+1)})$
$\Rightarrow j_1 - i + 4 = 2(m + n) + 1 - j_2 \Rightarrow j_2 = 2m + n + i - 4$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 23: For $i = 1, 2$ and $j = n + 1$, assume that $f_1(u_{(i_1+1) j} u_{i_1 j}) = f_1(u_{i_2 1} u_{(i_2+1) 1})$
$\Rightarrow j - i + 4 = i \Rightarrow n + 5 = 2i$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 24: For $i_1 = 1,2$, $j_1 = n+ 1$, $i_2 = 1$ and j_2 in $1 \le j \le n$, assume that $f_1(u_{(i_1+1) j_1} u_{i_1 j_1}) = f_1(u_{(i_2+1) j_2} u_{i_2 (j_2+1)})$
$\Rightarrow j_1 - i_1 + 4 = 2(m + n + j_2) - 1 \Rightarrow 6 = 2(m + j_2) + n + i_1$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 25: For $i_1 = 1,2$, $j_1 = n+ 1$, $i_2 = 1$ and j_2 in $1 \le j \le n$, assume that $f_1(u_{(i_1+1) j_1} u_{i_1 j_1}) = f_1(u_{(i_2+1)(j_2+1)} u_{i_2 j_2})$
$\Rightarrow j_1 - i_1 + 4 = 2(m + n + j_2) \Rightarrow 5 = 2(m + j_2) + n + i_1$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 26: For $i_1 = 1,2$, $i_2 = 2$, and j in $1 \le j \le n$, assume that $f_1(u_{(i_1+1) j} u_{i_1 j}) = f_1(u_{(i_2+1) j} u_{i_2 (j+1)})$
$\Rightarrow j - i_1 + 4 = 2(m + 2n + j) - 1 \Rightarrow 5 = 2(m + 2n) + j + i_1$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 27: For $i_1 = 1,2$, $i_2 = 2$ and j in $1 \le j \le n$, assume that $f_1(u_{(i+1) j} u_{ij}) = f_1(u_{(i+1)(j+1)} u_{ij})$
$\Rightarrow j - i_1 + 4 = 2(m + 2n + j) \Rightarrow 4 = 2(m + 2n) + j + i_1$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 28: For $i_1 = 1,2$, $i_2 = 2$ and j in $1 \le j \le n$, assume that $f_1(u_{(i+1) j} u_{ij}) = f_1(u_{ij} u_{i(j+1)})$
$\Rightarrow j - i_1 + 4 = 4mn + i_2 + j \Rightarrow 2 = 4mn + i_1$, a contradiction
$\Rightarrow$the edge labels are distinct.

Case – 29: For $i_1 = 1,2$, $i_2 = 1$, j_1 in $1 \le j \le n$ and j_2 in $2 \le j \le n$ assume that $f_1(u_{(i_1+1) j_1} u_{i_1 j_1}) = f_1(u_{i_2 j_2} u_{(i_2+1) j_2})$
$\Rightarrow j_1 - i_1 + 4 = 2mn + 2m + 3n + 1 - j_2 \Rightarrow j_1 + j_2 = 2mn + 2m + 3n + i_1 - 3$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 30: For $i_1 = 1,2$, $i_2 = 2$, j_1 in $1 \le j \le n$ and j_2 in $2 \le j \le n$, assume that $f_1(u_{(i_1+1) j_1} u_{i_1 j_1}) = f_1(u_{i_2 j_2} u_{(i_2+1) j_2})$
$\Rightarrow j_1 - i_1 + 4 = 4(mn + 1) - j_2 \Rightarrow j_1 + j_2 = 4mn + i_1$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 31: For $i = 1,2$ and j in $1 \le j \le n$, assume that $f_1(u_{1j} u_{1(j+1)}) = f_1(u_{i1} u_{(i+1)1})$
$\Rightarrow 2(m + n) + 1 - j = i \Rightarrow 2(m + n) + 1 = i + j$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 32: For $i = 1$ and j in $1 \le j \le n$, assume that $f_1(u_{1j} u_{1(j+1)}) = f_1(u_{(i+1) j} u_{i(j+1)})$
$\Rightarrow 2(m + n) + 1 - j = 2(m + n + j) - 1 \Rightarrow 2 = 3j$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 33: For $i = 1$ and j in $1 \le j \le n$, assume that $f_1(u_{1j} u_{1(j+1)}) = f_1(u_{(i+1)(j+1)} u_{ij})$
$\Rightarrow 2(m + n) + 1 - j = 2(m + n + j) \Rightarrow 3j = 1$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 34: For $i = 2$ and j in $1 \le j \le n$, assume that $f_1(u_{1j} u_{1(j+1)}) = f_1(u_{(i+1) j} u_{i(j+1)})$
$\Rightarrow 2(m + n) + 1 - j = 2(m + 2n + j) - 1 \Rightarrow 2 = 2n + 3j$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 35: For $i = 2$ and j in $1 \le j \le n$, assume that $f_1(u_{1j} u_{1(j+1)}) = f_1(u_{(i+1)(j+1)} u_{ji})$
$\Rightarrow 2(m + n) + 1 - j = 2(m + 2n + j) \Rightarrow 1 = 2n + 3j$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 36: For $i = 2$ and j in $1 \le j \le n$, assume that $f_1(u_{1j} u_{1(j+1)}) = f_1(u_{ij} u_{i(j+1)})$,
$\Rightarrow 2(m + n) + 1 - j = 4mn + i + j \Rightarrow 2(m + n) + 1 = 4mn + 2j + 1$, a contradiction.
$\Rightarrow$the edge labels are all distinct.

Case – 37: For $i = 1$, j_1 in $1 \le j \le n$ and j_2 in $2 \le j \le n$, assume that = $f_1(u_{1 j_1} u_{1(j_1+1)}) = f_1(u_{i j_2} u_{(i+1) j_2})$
$\Rightarrow 2(m + n) + 1 - j_1 = 2mn + 2m + 3n + 1 - j_2 \Rightarrow j_2 - j_1 = 2mn + n$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 38: For $i = 2$, j_1 in $1 \le j \le n$ and j_2 in $2 \le j \le n$, assume that $f_1\left(u_{1j_1}u_{1(j_1+1)}\right) = f_1(u_{ij_2}u_{(i+1)j_2})$
$\Rightarrow 2(m + n) + 1 - j_1 = 4(mn + 1) - j_2 \Rightarrow j_2 - j_1 = 4mn - 2(m + n) + 3$, a contradiction
$\Rightarrow$the edge labels are distinct.

Case – 39: For $i_1 = 1, 2$, $i_2 = 1$ and j in $1 \le j \le n$, assume that $f_1\left(u_{i1}u_{(i+1)1}\right) = f_1\left(u_{(i+1)j}u_{i(j+1)}\right)$
$\Rightarrow i_1 = 2(m + n + j) - 1 \Rightarrow i_1 + 1 = 2(m + n + j)$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 40: For $i_1 = 1, 2$, $i_2 = 1$ and j in $1 \le j \le n$, assume that $f_1\left(u_{i1}u_{(i+1)1}\right) = f_1\left(u_{(i+1)(j+1)}u_{ij}\right)$
$\Rightarrow i_1 = 2(m + n + j)$, a contradiction.
$\Rightarrow$the edge labels are all distinct.

Case – 41: For $i_1 = 1, 2$, $i_2 = 2$ and j in $1 \le j \le n$, assume that $f_1\left(u_{i_11}u_{(i_1+1)1}\right) = f_1(u_{(i_2+1)j}u_{i_2(j+1)})$
$\Rightarrow i_1 = 2(m + 2n + j) - 1 \Rightarrow i_1 + 1 = 2(m + 2n + j)$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 42: For $i_1 = 1, 2$, $i_2 = 2$ and j in $1 \le j \le n$, assume that $f_1\left(u_{i_11}u_{(i_1+1)1}\right) = f_1(u_{(i_2+1)(j+1)}u_{i_2j})$
$\Rightarrow i_1 = 2(m + 2n + j)$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 43: For $i_1 = 1, 2$, $i_2 = 2$ and j in $1 \le j \le n$, assume that $f_1\left(u_{i_11}u_{(i_1+1)1}\right) = f_1(u_{i_2j}u_{i_2(j+1)})$
$\Rightarrow i_1 = 4mn + i_2 + j \Rightarrow i_1 = 4mn + 2 + j$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 44: For $i_1 = 1, 2$, $i_2 = 1$ and j in $2 \le j \le n$, assume that $f_1\left(u_{i1}u_{(i+1)1}\right) = f_1\left(u_{ij}u_{(i+1)j}\right)$
$\Rightarrow i_1 = 2mn + 2m + 3n + 1 - j \Rightarrow i_1 + j = 2mn + 2m + 3n + 1$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 45: For $i_1 = 1, 2$, $i_2 = 2$ and j in $2 \le j \le n$, assume that $f_1\left(u_{i_11}u_{(i_1+1)1}\right) = f_1(u_{i_2j}u_{(i_2+1)j})$
$\Rightarrow i_1 = 4(mn + 1) - j \Rightarrow i_1 + j = 4(mn + 1)$, a contradiction
$\Rightarrow$the edge labels are distinct.

Case – 46: For $i = 1$ and j in $1 \le j \le n$, assume that $f_1\left(u_{(i+1)j}u_{i(j+1)}\right) = f_1\left(u_{(i+1)(j+1)}u_{ij}\right)$
$\Rightarrow 2(m + n + j) - 1 = 2(m + n + j)$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 47: For $i_1 = 1$, $i_2 = 2$ and j in $1 \le j \le n$, assume that $f_1\left(u_{(i_1+1)j}u_{i_1(j+1)}\right) = f_1(u_{(i_2+1)j}u_{i_2(j+1)})$
$\Rightarrow 2(m + n + j) - 1 = 2(m + 2n + j) - 1 \Rightarrow 1 = 2$, absurd
$\Rightarrow$the edge labels are all distinct.

Case – 48: For $i_1 = 1$, $i_2 = 2$ and j in $1 \le j \le n$, assume that $f_1\left(u_{(i_1+1)j}u_{i_1(j+1)}\right) = f_1(u_{(i_2+1)(j+1)}u_{i_2j})$
$\Rightarrow 2(m + n + j) - 1 = 2(m + 2n + j) \Rightarrow 2n = -1$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 49: For $i_1 = 1$, $i_2 = 2$ and j in $1 \le j \le n$, assume that $f_1\left(u_{(i_1+1)j}u_{i_1(j+1)}\right) = f_1(u_{i_2j}u_{i_2(j+1)})$
$\Rightarrow 2(m + n + j) - 1 = 4mn + i_2 + j \Rightarrow 2m + 2n + j = 4mn + 3$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 50: For $i = 1$, j_1 in $1 \le j \le n$ and j_2 in $2 \le j \le n$, assume that $f_1\left(u_{(i+1)j_1}u_{ij_1}\right) = f_1(u_{ij_2}u_{(i+1)j_2})$
$\Rightarrow 2(m + n + j_1) - 1 = 2mn + 2m + 3n + 1 - j_2 \Rightarrow 2j_1 + j_2 = 2mn + n + 2$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 51: For $i_1 = 1$, $i_2 = 2$, j_1 in $1 \le j \le n$ and j_2 in $2 \le j \le n$, assume that $f_1\left(u_{(i_1+1)j_1}u_{i_1j_1}\right) = f_1(u_{i_2j_2}u_{(i_2+1)j_2})$
$\Rightarrow 2(m + n + j_1) - 1 = 4(mn + 1) - j_2 \Rightarrow 2(m + n + j_1) + j_2 = 4mn + 5$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 52: For $i_1 = 1$, $i_2 = 2$ and j in $1 \le j \le n$, assume that $f_1\left(u_{(i_1+1)(j+1)}u_{i_1j}\right) = f_1(u_{(i_2+1)j}u_{i_2(j+1)})$
$\Rightarrow 2(m + n + j) = 2(m + 2n + j) - 1 \Rightarrow 1 = 2n$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 53: For $i_1 = 1$, $i_2 = 2$ and j in $1 \le j \le n$ assume that $f_1\left(u_{(i_1+1)(j+1)}u_{i_1j}\right) = f_1(u_{(i_2+1)(j+1)}u_{i_2j})$
$\Rightarrow 2(m + n + j) = 2(m + 2n + j) \Rightarrow n = 0$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 54: For $i_1 = 1$, $i_2 = 2$ and j in $1 \le j \le n$, assume that $f_1\left(u_{(i_1+1)(j+1)}u_{i_1j}\right) = f_1(u_{i_2j}u_{i_2(j+1)})$
$\Rightarrow 2(m + n + j) = 4mn + i_2 + j \Rightarrow 2(m + n) + j = 4mn + 2$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 55: For $i = 1$, j_1 in $1 \le j \le n$ and j_2 in $2 \le j \le n$, assume that $f_1\big(u_{(i+1)(j_1+1)}u_{ij_1}\big) = f_1(u_{ij_2}u_{(i+1)j_2})$
$\Rightarrow 2(m + n + j_1) = 2mn + 2m + 3n + 1 - j_2 \Rightarrow 2j_1 + j_2 = 2mn + n + 1$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 56: For $i_1 = 1$, $i_2 = 2$, j_1 in $1 \le j \le n$ and j_2 in $2 \le j \le n - 1$, assume that $f_1\big(u_{(i_1+1)(j_1+1)}u_{i_1j_1}\big) = f_1(u_{i_2j_2}u_{(i_2+1)j_2})$
$\Rightarrow 2(m + n + j_1) = 4(mn + 1) - j_2 \Rightarrow 2(m + n + j_1) + j_2 = 4(mn + 1)$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 57: For $i = 2$ and j in $1 \le j \le n$, assume that $f_1\big(u_{(i+1)j}u_{i(j+1)}\big) = f_1(u_{(i+1)(j+1)}u_{ij})$
$\Rightarrow 2(m + 2n + j) - 1 = 2(m + 2n + j)$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 58: For $i = 2$ and j in $1 \le j \le n$ and j in $1 \le j \le n$, assume that $f_1\big(u_{(i+1)j}u_{i(j+1)}\big) = f_1(u_{ij}u_{i(j+1)})$
$\Rightarrow 2(m + 2n + j) - 1 = 4mn + i + j \Rightarrow 2m + 4n + j = 4mn + 3$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 59: For $i_1 = 1$, $i_2 = 2$, j_1 in $1 \le j \le n$ and j_2 in $2 \le j \le n - 1$, assume that $f_1\big(u_{(i_1+1)j_1}u_{i_1(j_1+1)}\big) = f_1(u_{i_2j_2}u_{(i_2+1)j_2})$
$\Rightarrow 2(m + 2n + j_1) - 1 = 2mn + 2m + 3n + 1 - j_2 \Rightarrow n + 2j_1 + j_2 = 2(mn + 1)$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 60: For some $i = 2$, j_1 in $1 \le j \le n$ and j_2 in $2 \le j \le n$, assume that $f_1\big(u_{(i+1)j_1}u_{i(j_1+1)}\big) = f_1(u_{ij_2}u_{(i+1)j_2})$
$\Rightarrow 2(m + 2n + j_1) - 1 = 4(mn + 1) - j_2 \Rightarrow 2(m + 2n + j_1) + j_2 = 4mn + 5$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 61: For $i = 2$ and j in $1 \le j \le n$, assume that $f_1\big(u_{(i+1)(j+1)}u_{ij}\big) = f_1\big(u_{ij}u_{i(j+1)}\big)$
$\Rightarrow 2(m + 2n + j) = 4mn + i + j \Rightarrow 2(m + 2n) + j = 4mn + 2$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 62: For $i_1 = 2$, $i_2 = 1$, j_1 in $1 \le j \le n$, and j_2 in $2 \le j \le n$, assume that $f_1\big(u_{(i_1+1)(j_1+1)}u_{i_1j_1}\big) = f_1(u_{i_2j_2}u_{(i_2+1)j_2})$
$\Rightarrow 2(m + 2n + j_1) = 2mn + 2m + 3n + 1 - j_2 \Rightarrow n + 2j_1 + j_2 = 2mn + 1$, a contradiction
$\Rightarrow$the edge labels are all distinct.

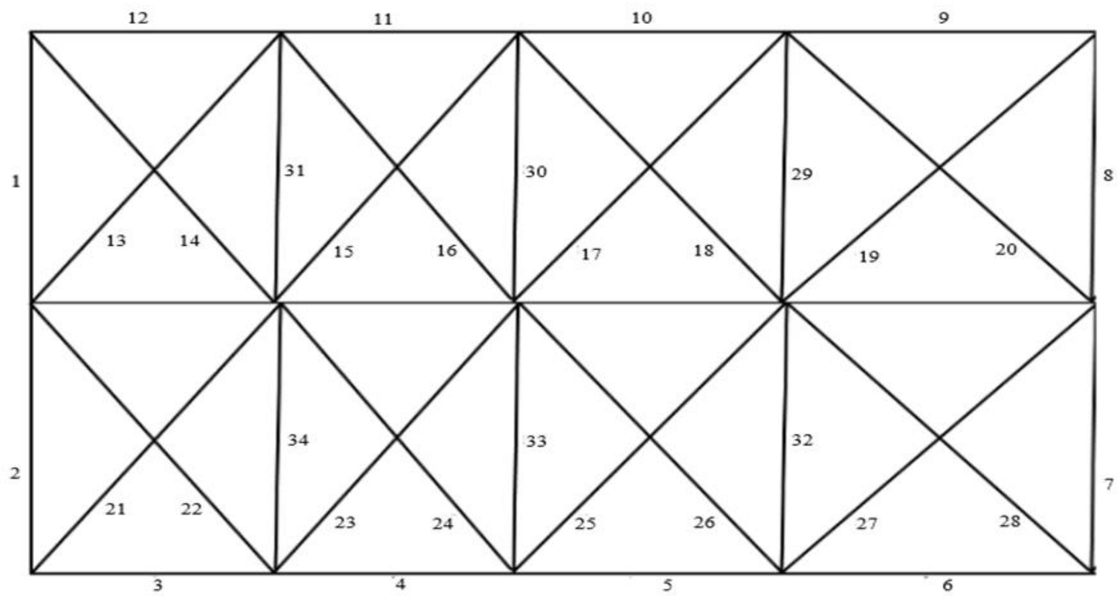
Case – 63: For some $i = 2$, j_1 in $1 \le j \le n$, and j_2 in $2 \le j \le n$, assume that $f_1\big(u_{(i+1)(j_1+1)}u_{ij_1}\big) = f_1(u_{ij_2}u_{(i+1)j_2})$
$\Rightarrow 2(m + 2n + j_1) = 4(mn + 1) - j_2 \Rightarrow 2(m + 2n + j_1) + j_2 = 4(mn + 1)$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 64: For some $i_1 = 2$, j_1 in $1 \le j \le n$, $i_2 = 1$ and j_2 in $2 \le j \le n$, assume that $f_1(u_{i_1j_1}u_{i_1(j_1+1)}) = f_1(u_{i_2j_2}u_{(i_2+1)j_2})$
$\Rightarrow 4mn + i_1 + j_1 = 2mn + 2m + 3n + 1 - j_2 \Rightarrow 2mn + j_1 + j_2 + 1 = 2m + 3n$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 65: For $i = 2$, j_1 in $1 \le j \le n$, and j_2 in $2 \le j \le n - 1$, assume that $f_1(u_{ij_1}u_{i(j_1+1)}) = f_1(u_{ij_2}u_{(i+1)j_2})$
$\Rightarrow 4mn + i + j_1 = 4(mn + 1) - j_2 \Rightarrow j_1 + j_2 = 2$, a contradiction
$\Rightarrow$the edge labels are all distinct.

Case – 66: For some $i_1 = 1$, $i_2 = 2$ and j in $2 \le j \le n$, assume that $f_1(u_{i_1j}u_{(i_1+1)j}) = f_1(u_{i_2j}u_{(i_2+1)j})$
$\Rightarrow 2mn + 2m + 3n + 1 - j = 4(mn + 1) - j \Rightarrow 2m + 3n = 2mn + 3$, a contradiction
$\Rightarrow$the edge labels are all distinct.

From the cases discussed above, it follows that the edge labels are all distinct. The label of a vertex is the sum of labels of the edges that are incident with it. Since the edge labels are all distinct, it follows that the vertex labels are all distinct. Hence the graph $T(2,n)$, $n \ge 3$ admits vertex antimagic edge labeling. As an illustration, we consider the graph of $T(2,n)$, $n \ge 3$.



Tessellation of K_4 T(2,4)

5. CONCLUSION

In this paper, we have proved that the tessellation of K_4, T(m,n) admits vertex antimagic edge labeling for $m = 1,2$ and $n \geq 3$. The admittance of antimagic labeling has been proved using two theorems with multiple cases. The same can be used to prove for T(m, n), $m \geq 3$.

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