RELIABILITY ANALYSIS FOR BREAKDOWN AND RECOVERY POLICIES OF A MARKOVIAN FINITE CAPACITY QUEUEING SYSTEM

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ABSTRACT

In this paper the Reliability and Availability analysis of a finite capacity Markovian queueing model is established. The breakdown and recovery policies of the transient state queueing system are formulated using Ordinary Differential-Difference equations. Based on the time-dependent analysis of the system, the reliability and availability functions are considered. A special case for N=4 is employed and the system of equations are solved to find the transient probabilities by Fourth-Order Runge-Kutta numerical method. The model is illustrated numerically and shown graphically.

KEYWORDS

Availability, Breakdown, Markovian Queue, Recovery policies, Reliability, Runge-Kutta.

1. INTRODUCTION

The Stochastic process models that is being applied in the real world applications, Markov chain has a special place which is characterized by the Markov property that is “Memory-less” property. The basic assumption of a Markov chain is that probability of transitioning to any particular state depends on only the current state and the time elapsed. The Continuous time Markov Chain (CTMC) consists of finite states space which is characterized by the transition rates with respect to time of transition probabilities between the states that runs through continuous time. M.A.El-Damcese and N.S.Temraz [1] formulated the study on Markov models for analysis of reliability and availability functions for parallel repairable system subject to three types of failure rates (i.e.,human, hardware,software) and also thecommon cause for failure rate.

The CTMC is widely used for Queueing theory to find the solution for some of the simple basic models in Queueing. In general, the queueing model is assumed to work at constant speed which may not be correct always since the state of the system may affect the productivity of the server. Many traditional studies have been conducted to analyze the steady state solution of the models. The results which are obtained from the steady-state are used to study the performance measures of the system on a long time period whereas the transient analysis is used to understand the behavior of the system. Some types of methods have been used to find the time dependent system size probabilities of a queueing system. Bailey [2] obtained the solution for the transient analysis of the M/M/1 queueing model using generating function method.

The Breakdown and Recovery policies are introduced into the Markovian Queueing model leads its way to the study of the System Reliability. The Reliability of a System is defined as the probability that the device will perform the required function during a specific period of time and under stated conditions. This may be expressed as,

$$R(t) = \Pr\{T>t\} = \int_t^{\infty} f(x)\,dx$$

Where, f(x) is the failure probability density function and t is the length of the period of time. The system reliability depends not only on the reliabilities of components but also on their dependencies among them. Availability is the probability that a system is not failed or undergoing repair action when it needs to be used. Availability of a system is typically measured as a factor of its reliability – as reliability increases, so does availability. Limiting (or steady-state) availability is represented by:

$$A(\infty) = \lim_{t\to\infty} A(t)$$

Queueing models subject to breakdowns and recovery policies are studied along with arrival and service time either from the queueing theory point of view or of reliability theory. S.N.Singh and S.K.Singh [3] analyzed the two unit parallel system with Erlangian repair time and found the solutions for the Reliability and Availability functions. S.Sophia [4] made a study on the transient analysis of M/M/1 queue using chain sequence along with catastrophes, failures and repairs. Nneamaka Judith Ezeagu [5] analysed the finite capacity of M/M/1 queueing models with breakdowns and two types of recovery policies. Yang et al. [6] analysed a finite capacity M/M/1 system with a threshold-based recovery policy under the steady state condition.

The aim of this paper is to find the Reliability and Availability analysis of the finite capacity Markovian queueing system with breakdown and recovery policies. A special case for N=4 is employed and the system of differential-difference equations are solved for time-dependent probabilities by Fourth-Order Runge-Kutta numerical method. The Reliability and Availability of the model is illustrated numerically and shown graphically.

2. DESCRIPTION OF THE MODEL

Consider a Markovian finite capacity single server queueing system of $N<\infty$ with working breakdowns and recovery states. It is been assumed that each machines arrive independently following a Poisson process with constant parameter λ (arrival rate). These machines are automatically arriving to the service station in which the service follows a FCFS discipline. Let S (t) denotes the state of the server at any instant of time t such that:

$$S(t) = \begin{cases} 1, & if\ the\ server\ is\ in\ working\ state \\ 2, & if\ the\ server\ is\ in\ breakdown\ state \end{cases}$$

Therefore, the server follows a Markovian environmental process $\{S(t), t\geq 0\}$ on a finite state space {1, 2}. The service station starts at the rate μ_1 for the normal working period which is assumed to be exponentially distributed. μ_2 is the service station rate when the system is in breakdown state, is also assumed to be exponentially distributed. When the service station is in the breakdown state it means that it will still provide service but at a lower rate.

Apart from the arrival and the service processes when the system is not empty breakdowns also occur at the service facility with the breakdown rate α. Therefore whenever the station is in the breakdown state it is immediately recovered in the recovery state with rate β. The breakdown and recovery times are considered to be exponentially distributed. During the breakdown situation the service occurs at a slow rate. Once the service station recovers to normal situation it serves the customer on the normal service rate.

Let N (t) denote the total number of customers in the system at any time t. Thus the system describes a two-dimensional continuous time markov chain(CTMC) $\{S(t),N(t), t\geq 0\}$ on the state space ((i, n): $i\in\{0,1\}$, n = {0,1,2,…,N}. The time-dependent state probability is defined as

$P_{i,n}(t)$=P[S(t)=I, N(t)=n]; ∀ i=0,1; n={0,1,…,N}

where, i represent the probability for each server state, that there are n customers in the system at any time t. The State Transition diagram for the Markovian queueing model with finite capacity is given in Figure 1.

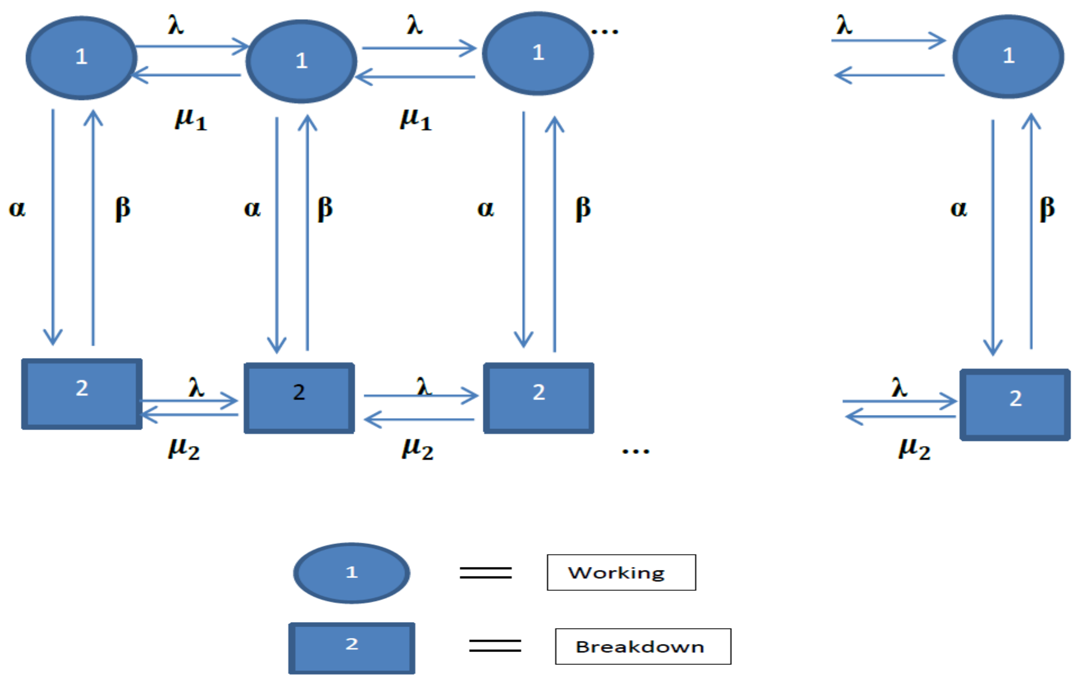


Figure 1: State Transition Diagram for Markovian queueing model with finite capacity

The state probabilities $P_n(t)$,n=0, 1, 2… N for the system can be obtained by the Chapman-Kolmogorov forward differential-difference equations with breakdown and recovery policies as follows:

$$\frac{dP_{1,0}}{dt} = -\lambda P_{1,0}(t) + \mu P_{1,1}(t) + \beta P_{2,0}(t) \quad (1)$$

$$\frac{dP_{1,n}(t)}{dt} = \lambda P_{1,n-1}(t) - (\lambda + \mu_1 + \alpha) P_{1,n}(t) + \mu_1 P_{1,n+1}(t) + \beta P_{2,n}(t) \quad (2)$$

$$\frac{dP_{1,N}(t)}{dt} = \lambda P_{1,N-1}(t) - (\mu_1 + \alpha) P_{1,N}(t) + \beta P_{2,N}(t) \quad (3)$$

$\frac{dP_{2,0}(t)}{dt}$=-(λ+β)$P_{2,0}(t)$+$\mu_2 P_{2,1}(t)$ (4)

$\frac{dP_{2,n}(t)}{dt}$= λ $P_{2,n-1}(t)$-(λ + $\mu_2 + \alpha$) $P_{2,n}(t)$+$\mu_2 P_{2,n+1}(t)$+α$P_{1,n}(t)$ (5)

$\frac{dP_{2,N}(t)}{dt}$= $\lambda P_{2,N-1}(t)$-($\mu_2 + \beta$)$P_{2,N}(t)$+$\alpha P_{1,N}(t)$ (6)

without loss of generality the initial state probabilities are given by

$P_{1,0}(0)$=1, $P_{1,n}(0)$, ∀ n=1,2…,N;

$P_{2,n}(0)$=0, ∀ n=0, 1, 2… N.

where,

$P_{1,N}$ − The probability that there are n customers in the system when the service station is in the normal working period

$P_{2,N}$ −The probability that there are n customers in the system when the service station is in breakdown period

The above system of equations (1)-(6) can be solved for transient state probabilities. These time-dependent probabilities can be used to find the Reliability and Availability of the system.

3. SPECIAL CASE

Consider the finite space of Markovian Queueing model as N=4, the above system of equations (1)-(6), are obtained as

$\frac{dP_{1,0}}{dt}$=$-\lambda$ $P_{1,0}(t)$+ μ_1 $P_{1,1}(t)$+β$P_{2,0}(t)$ (7)

$$\frac{dP_{1,1}(t)}{dt}= \lambda\, P_{1,n-1}(t)-(\lambda + \mu_1 + \alpha)\, P_{1,n}(t)+\mu_1 P_{1,n+1}(t)+\beta P_{2,1}(t) \quad (8)$$

$$\frac{dP_{1,2}(t)}{dt}= \lambda\, P_{1,1}(t)-(\lambda + \mu_1 + \alpha)\, P_{1,2}(t)+\mu_1 P_{1,3}(t)+\beta P_{2,2}(t) \quad (9)$$

$$\frac{dP_{1,3}(t)}{dt}= \lambda\, P_{1,2}(t)-(\lambda + \mu_1 + \alpha)\, P_{1,3}(t)+\mu_1 P_{1,4}(t)+\beta P_{2,3}(t) \quad (10)$$

$$\frac{dP_{1,4}(t)}{dt}=\lambda P_{1,3}(t)-(\mu_1 + \alpha)P_{1,4}(t)+\beta P_{2,4}(t) \quad (11)$$

$$\frac{dP_{2,0}(t)}{dt}=-(\lambda+\beta)P_{2,0}(t)+\mu_2 P_{2,1}(t) \quad (12)$$

$$\frac{dP_{2,1}(t)}{dt}= \lambda\, P_{2,0}(t)-(\lambda + \mu_2 + \alpha)\, P_{2,1}(t)+\mu_2 P_{2,2}(t)+\alpha P_{1,1}(t) \quad (13)$$

$$\frac{dP_{2,2}(t)}{dt}= \lambda\, P_{2,1}(t)-(\lambda + \mu_2 + \alpha)\, P_{2,2}(t)+\mu_2 P_{2,3}(t)+\alpha P_{1,2}(t) \quad (14)$$

$$\frac{dP_{2,3}(t)}{dt}= \lambda\, P_{2,2}(t)-(\lambda + \mu_2 + \alpha)\, P_{2,3}(t)+\mu_2 P_{2,4}(t)+\alpha P_{1,3}(t) \quad (15)$$

$$\frac{dP_{2,4}(t)}{dt}= \lambda P_{2,3}(t)-(\mu_2 + \beta)P_{2,4}(t)+\alpha P_{1,4}(t) \quad (16)$$

The system of equations (7)-(16) are solved by Fourth-Order Runge-Kutta method and the transient probabilities can be obtained.

The Reliability and Availability of the system can also be obtained by using time-dependent probabilities.

3.1. NUMERICAL ILLUSTRATION

The prime focus of this paper is to analyze the transient behavior of the Markovian finite capacity queueing system which is modeled by the differential difference equations. Assuming the time range from t=0 to t=130, and the standard parameter values as λ=0.30,

μ_1=0.6, μ_2=0.1, α=0.10, β=0.2. The system of equations (7)-(16) are solved for time-dependent probabilities. $P_n(t)$, the probability of n customers in the system using Fourth order Runge-Kutta numerical method. A few probability curves are displayed in Figure 2 which shows the distribution trend of the system probabilities over the specified time intervals. The Reliability and Availability of the system are also analyzed and are shown in Figures 3 and 4 respectively.

Figure 2: Time-Dependent Probabilities of the Markovian Queueing System

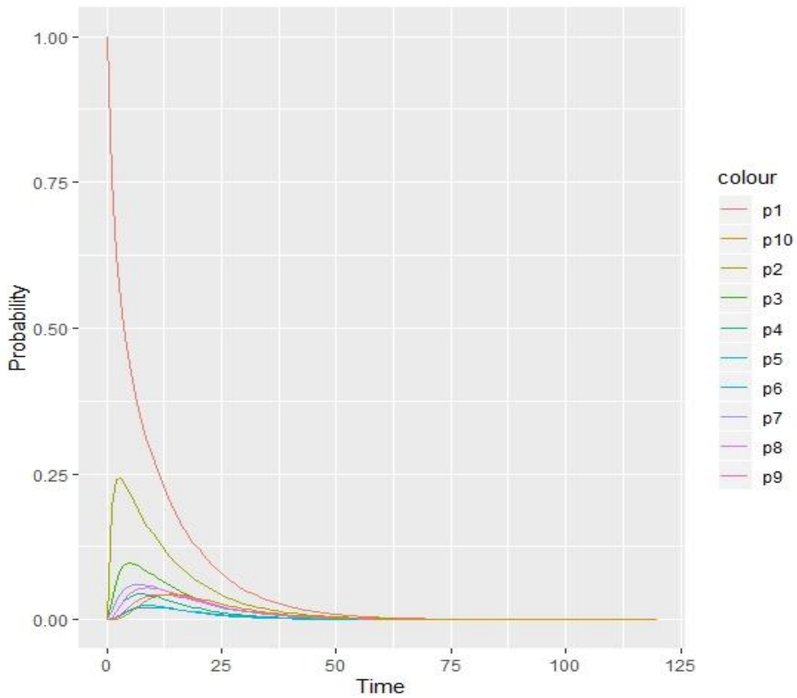


Figure 3 represents the Reliability function of the system. The graph for R (t) has been plotted with respect to time. It is observed from the graph that the Reliability decreases as time grows, a note has been made that as time increases, R (t) initially decreases and after some time it becomes almost constant.

Figure 3: System Reliability

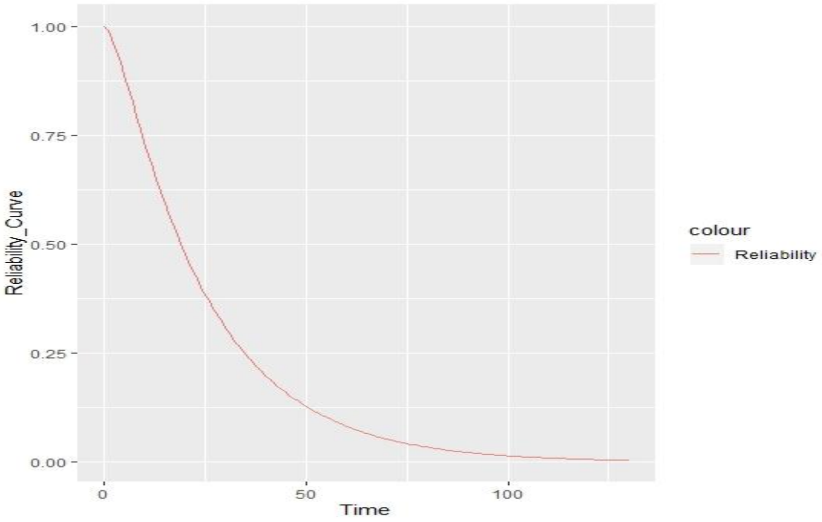
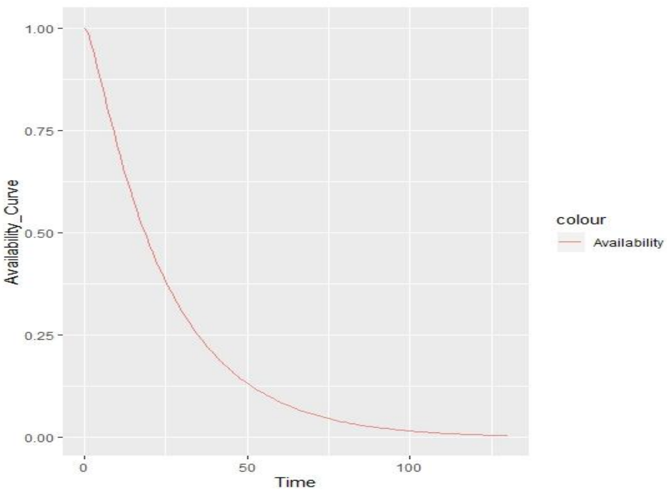


Figure 4 represents the Availability function of the system. The graph for A (t) has been plotted with respect to time. It is observed from the graph that the Availability decreases as time grows, a note has been made that as time increases, A (t) initially decreases and after some time it becomes almost constant.

Figure 3 Availability curve



4. CONCLUSION

The Reliability and Availability analysis of the time-dependent finite capacity Markovian queueing system with breakdown and recovery polices are studied in this paper. The state probabilities for the transient system are obtained for the general case by differential-difference equations. A special case for N=4 is employed and are solved by using Fourth-Order Runge-Kutta method. The Reliability and Availability analysis are illustrated numerically and shown graphically. It is found that as time increases both Availability and Reliability decreases.

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