On Distance and Similarity Measures Over Intuitionistic Fuzzy MultiSets of Second Type

K. Mariyam Jameela* and R. Srinivasan

* Ph.D Full Time Research Scholar, Department of Mathematics,
Islamiah College (Autonomous),
Vaniyambadi 635 752, India

Department of Mathematics, Islamiah College (Autonomous),
Vaniyambadi 635 752, India

*e-mail: srinivasanmaths@yahoo.com

Abstract

In this paper, the various distance measures of IFMSST is defined. The measures are based on Hausdorff distance, Hamming distance and normalized Hamming distance. Further, the comparison is made between the measures of IFMS and IFMSST to get the shortest distance.

AMS Subject Classification: 03E72

Keywords: Fuzzy Sets (FS), Intuitionistic Fuzzy Sets (IFS), Multi sets (MS), Intuitionistic Fuzzy Multisets (IFMS).

1. Introduction

Modern set theory formulated by the German mathematician George Cantor is fundamental for the whole mathematics. In fact, set theory is the language of mathematics, science, logic and philosophy. One issue associated with the notion of set is the concept of vagueness.

Considering the unpredictable factor in decision making Lofti A Zadeh introduced the idea of Fuzzy set which has a membership function that assigns to each element of the universe of discourse, a member from the unit interval [0; 1] to indicate the degree of belongingness to the set under consideration. Atanassov subsequently proposed the concept of Intuitionistic Fuzzy Set by bringing a non-membership function together with the membership function of the fuzzy set. Among the various notions of higher order fuzzy sets, IFS proposed by Atanassov provides a flexible framework to elaborate uncertainty and vagueness.

As a generalization of fuzzy sets, Yager introduced the concept of Fuzzy Multiset. An element of a Fuzzy Multiset can occur more than once with possibly the same or different membership values. Then years after, Shinoj and Sunil made an attempt to combine the concepts and named it Intuitionistic Fuzzy Multi Set.
The present authors have introduced Intuitionistic Fuzzy Multisets of Second type which is a further extentension of IFMS.

This paper proceeds as follows: In section 2, we give some basic definitions related to fuzzy set, Intuitionistic fuzzy set, Fuzzy multi set and Intuitionistic fuzzy multi set. In subsequent sections; the distance measures are defined for IFMS and IFMSST and also the measures are compared to get the shortest distance among them. The paper is concluded in section 5.

2. Preliminaries

**Definition 2.1** Let X be a non-empty set. A fuzzy set A drawn from X is defined as
\[ A = \{ x, \mu_A(x) : x \in X \} \]
where \( \mu_A(x) : X \rightarrow [0,1] \) is the membership function of the fuzzy set A.

**Definition 2.2** Let X be a non-empty set. A fuzzy multiset (FMS) A drawn from X is characterized by a function count membership of A denoted by \( CM_A \) such that
\[ CM_A : X \rightarrow Q \]
where Q is the set of all crisp multisets drawn from the unit interval [0,1], then for any \( x \in X \), the value \( CM_A(x) \) is a crisp multiset drawn from [0,1]. For each \( x \in X \), the membership sequence is defined as the decreasingly ordered sequence of elements in \( CM_A(x) \). It is denoted by \( \mu_A^1(x), \mu_A^2(x), ..., \mu_A^n(x) \) where \( \mu_A^1(x) \geq \mu_A^2(x) \geq ... \geq \mu_A^n(x) \)

**Definition 2.3** Let X be a nonempty set. An Intuitionistic Fuzzy Set (IFS) A is an object having the form
\[ A = \{ x, \mu_A(x), \nu_A(x) : x \in X \} \]
where the functions \( \mu_A : X \rightarrow [0,1] \) and \( \nu_A : X \rightarrow [0,1] \) define respectively the degree of membership and the degree of non-membership of the element \( x \in X \) to the set A with \( 0 \leq \mu_A(x) + \nu_A(x) \leq 1 \) for each \( x \in X \).

**Remark:** Every Fuzzy Set A on a nonempty set X is obviously an IFS having the form \( A = \{ x, \mu_A(x), 1 - \mu_A(x) : x \in X \} \)

**Definition 2.4** Let X be a nonempty set. An Intuitionistic Fuzzy Multiset A denoted by IFMS drawn from X is characterized by two functions count membership of A (\( CM_A \)) and count non membership of A (\( CN_A \)) given respectively by \( CM_A : X \rightarrow Q \) and \( CN_A : X \rightarrow Q \) where Q is the set of all crisp multisets drawn from the unit interval [0,1] such that for each \( x \in X \), the membership sequence is defined as a decreasingly ordered sequence of elements in which \( CM_A \) is denoted by \( \mu_A^1(x), \mu_A^2(x), ..., \mu_A^n(x) \) where \( \mu_A^1(x) \geq \mu_A^2(x) \geq ... \geq \mu_A^n(x) \) and the
corresponding non membership sequence will be denoted by \( \nu_{A_1}(x), \nu_{A_2}(x), \ldots, \nu_{A_n}(x) \) such that \( 0 \leq \mu_{A_i}(x) + \nu_{A_i}(x) \leq 1 \) for each \( x \in X \) and \( i = 1, 2, \ldots, n \).

**Remark.** Since we arrange the membership sequence in decreasing order the corresponding non membership sequence can be in increasing or decreasing order.

**Definition 2.5** Let \( X \) be a nonempty set. An Intuitionistic Fuzzy Multiset of second type \( A \) denoted by IFMSST drawn from \( X \) is characterized by two functions count membership of \( A \) (\( CM_A \)) and count non membership of \( A \) (\( CN_A \)) given respectively by \( CM_A : X \to Q \) and \( CN_A : X \to Q \) where \( Q \) is the set of all crisp multisets drawn from the unit interval \([0,1]\) such that for each \( x \in X \), the membership sequence is defined as a decreasingly ordered sequence of elements in which \( CM_A \) is denoted by \( \mu_1(x), \mu_2(x), \ldots, \mu_n(x) \) where \( \mu_1(x) \geq \mu_2(x) \geq \ldots \geq \mu_n(x) \) and the corresponding non membership sequence will be denoted by \( \nu_1(x), \nu_2(x), \ldots, \nu_n(x) \) such that \( 0 \leq (\mu_i(x))^2 + (\nu_i(x))^2 \leq 1 \) for each \( x \in X \) and \( i = 1, 2, \ldots, n \).

An IFMS of second type is denoted by \( A = \{ x, (\mu_1(x), \mu_2(x), \ldots, \mu_n(x)), (\nu_1(x), \nu_2(x), \ldots, \nu_n(x)) / x \in X \} \)

**Remark.** Since we arrange the membership function in decreasing order, the corresponding non membership sequence may not be in increasing or decreasing order.

**Definition 2.6** The degree of non determinacy (uncertainty or hesitancy ) of an element \( x \in X \) in the IFMSST \( A \) is defined by \( \pi_{A_i}(x) = \sqrt{1 - \mu_{A_i}(x)^2 - \nu_{A_i}(x)^2} \)

**Definition 2.7** Length of an element \( x \) in an IFMSST \( A \) is defined as the Cardinality of \( CM_A(x) \) or \( CN_A(x) \) for which \( 0 \leq (\mu_{A_i}(x))^2 + (\nu_{A_i}(x))^2 \leq 1 \) and it is denoted by \( L(x : A) \). That is \( L(x : A) = |CM_A(x)| = |CN_A(x)| \)

**Definition 2.8** If and \( B \) are IFMSST drawn from \( X \) then \( L(x : A,B) = \text{Max} \{ L(x : A), L(x : B) \} \) We can use the notation \( L(x) \) for \( L(x : A;B) \)
3. Distance Measures on IFMS

In this section, we present the Hausdorff distance, Hamming distance measure and the normalized Hamming distance measure for IFMS with suitable examples.

**Definition 3.1** Let $A$ and $B$ be any IFMS in $X$ defined as $A = \{ x, \mu^i_A(x), \nu^i_A(x) / x \in X \}$ and $B = \{ x, \mu^i_B(x), \nu^i_B(x) / x \in X \}$ for all $i = 1, 2, \ldots n$ then the distance measure $d(A,B)$ between the IFMS $A$ and $B$ is given by

**HAUSDORFF DISTANCE:**

$$d_{Hd}(A,B) = \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{\eta} \max \{ |\mu_A^i(x) - \mu_B^i(x)| + |\nu_A^i(x) - \nu_B^i(x)| + |\pi_A^i(x) - \pi_B^i(x)| \}$$

**HAMMING DISTANCE:**

$$d_H(A,B) = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{\eta} \{ |\mu_A^i(x) - \mu_B^i(x)| + |\nu_A^i(x) - \nu_B^i(x)| + |\pi_A^i(x) - \pi_B^i(x)| \}$$

**Normalized HAMMING DISTANCE:**

$$d_{n-H}(A,B) = \frac{1}{2n} \sum_{i=1}^{n} \sum_{j=1}^{\eta} \{ |\mu_A^i(x) - \mu_B^i(x)| + |\nu_A^i(x) - \nu_B^i(x)| + |\pi_A^i(x) - \pi_B^i(x)| \}$$

$$= \frac{1}{n} d_H(A,B)$$

**Example.** Let $X = \{x,y,z,w\}$ be any non empty set. Also let $A,B \in X$ be any IFMS defined as $A = \{ <x : (0.5,0.2,0.1) (0.3,0.2,0.2)>, <y : (0.6,0.4,0.3) (0.4,0.5,0.3)>, <w : (0,0,0) (1,1,1)> \}$

$B = \{ <x : (0.4,0.4,0.3) (0.6,0.5,0.5)>, <y : (0,0,0) (1,1,1)>, <z : (0.7,0.3,0.1) (0.3,0.2,0.1)> \}$

The Hausdorff distance between $A$ and $B$ is $d_{Hd}(A,B) = 0.4$

The Hamming distance between $A$ and $B$ is $d_H(A,B) = 5.5$
The normalized Hamming distance between $A$ and $B$ is $d_{n-H}(A,B) = 1.375$

It is clear from the above example that the Hausdorff distance gives the shortest measure.

4. Distance Measures on IFMSST

In this section, we define the Hausdorff distance, Hamming distance measure and the normalized Hamming distance measure for IFMSST with suitable examples.

Definition 4.1 Let $A$ and $B$ be any IFMSST in $X$ defined as $A = \{ x, \mu_A(x), v_A^i(x) / x \in X \}$ and $B = \{ x, \mu_B(x), v_B^i(x) / x \in X \}$ for all $i = 1, 2, \ldots n$ then the distance measure $d(A,B)$ between the IFMSST $A$ and $B$ is given by

HAUSDORFF DISTANCE:

$$d_{Hd}(A,B) = \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{\eta} \max\{ |\mu_A^i(x) - \mu_B^i(x)|^2, |v_A^i(x) - v_B^i(x)|^2, |\pi_A^i(x) - \pi_B^i(x)|^2 \}$$

HAMMING DISTANCE:

$$d_H(A,B) = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{\eta} \sqrt{ |\mu_A^i(x) - \mu_B^i(x)|^2 + |v_A^i(x) - v_B^i(x)|^2 + |\pi_A^i(x) - \pi_B^i(x)|^2 }$$

Normalized HAMMING DISTANCE:

$$d_{n-H}(A,B) = \frac{1}{2n} \sum_{i=1}^{n} \sum_{j=1}^{\eta} \sqrt{ |\mu_A^i(x) - \mu_B^i(x)|^2 + |v_A^i(x) - v_B^i(x)|^2 + |\pi_A^i(x) - \pi_B^i(x)|^2 } = \frac{1}{n} d_H(A,B)$$
Example. Let $X = \{x,y,z,w\}$ be any non empty set. Also let $A,B \in X$ be any IFMSST defined as

$$A = \{ <x : (0.5,0.2,0.1)(0.3,0.2,0.2)> <y : (0.6,0.4,0.3)(0.4,0.5,0.3)> <w : (0,0,0)(1,1,1)> \}$$

$$B = \{ <x : (0.4,0.4,0.3)(0.6,0.5,0.5)> <y : (0,0,0)(1,1,1)> <z : (0.7,0.3,0.1)(0.3,0.2,0.1)> \}$$

The Hausdorff distance between $A$ and $B$ is $d_{Hd}(A,B)= 0.289$
The Hamming distance between $A$ and $B$ is $d_H(A,B) = 1.486$
The normalized Hamming distance between $A$ and $B$ is $d_{n-H}(A,B) = 0.3715$
Thus the Hausdorff distance gives the best measure

Result.

As we compare the various distance measures, it is concluded that the hausdorff measure of IFMSST gives the shortest value than the others. As it has the good rate of confidence in terms of accuracy. Henceforth, we can apply the hausdorff measure of IFMSST in the real life problems to find the shortest distance.

<table>
<thead>
<tr>
<th>DISTANCE MEASURES</th>
<th>IFMS</th>
<th>IFMSST</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hausdorff distance</td>
<td>0.4</td>
<td>0.289</td>
</tr>
<tr>
<td>Hamming distance</td>
<td>5.5</td>
<td>1.486</td>
</tr>
<tr>
<td>normalized Hamming distance</td>
<td>1.375</td>
<td>0.3715</td>
</tr>
</tbody>
</table>
5. Conclusion

In this paper, we have defined various types over distance measures for IFMSST and the results are also compared. The proposed methods are mathematically valid and can be applied to any real life situations. There is an excellent opportunity for further research in this area.

References.

4. Mariyam Jameela K., Srinivasan R., A Study on IFMS of Type II IJMA 9(9),2018,10-13