Group Field Cosmology in $\mathcal{N} = 2$ Superspace

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In this paper, we analyse group field cosmology in $\mathcal{N} = 2$ superspace. The theory is discussed in a representation where a non-trivial mixing of the generators of supersymmetry occurs. Further, we derive an explicit expression for the superspace propagators in this superspace. Finally, these propagators are used for performing some perturbative calculations.

I. INTRODUCTION

The background independent loop quantum gravity in quantization of gravitational degrees of freedom has been widely investigated [1–3]. Basically, in this method the constraints are expressed in terms of the densitized triad and of the Ashtekar-Barbero connection [4–9]. The main challenges of such quantization method are the lack of complete definition of the quantum dynamics and the systematic proof that leads back the resulting theory to Einsteins gravity. For the sake of completeness of the definition of the quantum dynamics one has to embed the loop quantum gravity states into the larger framework of group field theories [10, 11] via spin-foam models [12, 13]. The group field theories are basically quantum field theories on group manifolds and the Feynman amplitudes of such theories are spin-foam models. The loop quantum gravity is a topology fixed object and therefore can be analysed by second quantized. However, the topology changing processes can not be analysed through second quantization approach and therefore one needs the third quantization for such processes [14–17]. The idea behind the third quantization formalism is to treat the multi-universe system as a quantum field theory on superspace [18]. Incidentally, the third quantization of loop quantum gravity naturally leads to the group field theory [19–22]. The minisuperspace (Wheeler-De Witt)

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approximations of group field theory leads to the group field cosmology [23]-[31]. We study the group field cosmology in $\mathcal{N}=2$ superspace. In this paper, we will extend this work and study a supersymmetric field theory with $\mathcal{N}=2$ superspace. In fact, we will analyse a theory where a non-trivial mixing of the original generators of supersymmetry occurs. We will also obtain explicit expression for propagators for this theory and use them for performing perturbative calculations.

This paper is organized as follows. In section 2, we discuss the main building block of supersymmetric group field cosmology. The theory is discussed in $\mathcal{N}=2$ superspace in section 3. Section 4 is devoted to study the superfields in superspace. The propagators are calculated in section 5 to analyze the perturbative calculations. We draw concluding remark in the last section.

II. SUPERSYMMETRIC GROUP FIELD COSMOLOGY

In this section, we study a model of quantum multiverse made up of homogeneous and isotropic universes filled up with a perfect fluid. We begin with the loop quantum cosmology with a massless scalar field ϕ as matter. The four-dimensional metric is then defined by

$$ds^2 = -N^2(t)dt^2 + a^2(t)\delta_{ab}dx^a dx^b, \tag{2.1}$$

where $N(t)$ and $a(t)$ are lapse function and scaler factor respectively. Here indices, a, b, denote the spatial indices. In loop quantum gravity the phase space is described by the Ashtekar-Barbero connection A_a^A and its canonically conjugate momentum, the densitized triad E_A^a, that plays the role of an "electric field". The capital alphabets, $A, B, ...$, are $SU(2)$ indices and label new degrees of freedom for the triad formulation. To define these objects, we introduce the co-triad e_a^A defined by $q_{ab} = e_a^A e_b^B \delta_{AB}$, where δ_{AB} is the Kronecker delta in three dimensions. Then the triad, e_A^a, is defined as its inverse $e_A^a e_b^B = \delta_b^a \delta_A^B$. The Ashtekar-Barbero connection is expressed by $A_a^A = \Gamma_a^A + \gamma K_a^A$, where γ is the Barbero-Immirzi parameter and K_a^A is the extrinsic curvature in triadic form, and Γ_a^A is the spin connection compatible with the densitized triad.

Now the eigenstates of the volume operator (finite cell) $\mathcal{V}$ with a basis, $|\nu\rangle$, are defined by $\mathcal{V}|\nu\rangle = 2\pi\gamma G|\nu||\nu\rangle$, where gravitational conguration variable $\nu = \pm a^2\mathcal{V}_0/2\pi\gamma G$ has the dimensions of length. The Hamiltonian constraint in the Plank units for a homogeneous isotropic universe is given by [43]

$$-B(\nu)[E^2 - \partial_\phi^2]\Phi(\nu,\phi) = K^2\Phi(\nu,\phi) = 0, \tag{2.2}$$

where $\Phi(\nu,\phi)$ is a wavefunction on configuration space and E^2 is a difference operator defined as

$$-E^2\Phi(\nu,\phi) = \frac{C^+(\nu)}{B(\nu)}\Phi(\nu+\nu_0,\phi) + \frac{C^0(\nu)}{B(\nu)}\Phi(\nu,\phi) + \frac{C^-(\nu)}{B(\nu)}\Phi(\nu-\nu_0,\phi), \tag{2.3}$$

and $K^2 = -B(\nu)[E^2 - \partial_\phi^2]$. Here ν_0 refers an elementary length unit, usually defined by the square root of the area gap and the functions $B(\nu), C^+(\nu), C^0(\nu)$ and $C^-(\nu)$ depend on the choice of the lapse function and on the particulars of quantization scheme used. For instance, in an improved dynamic scheme, these functions for particular choice of lapse function, i.e. $N = 1$, have following form:

$$\begin{aligned}
B(\nu) &= \frac{3\sqrt{2}}{8\sqrt{\sqrt{3}\pi\gamma G}}|\nu|\left|\left|\nu+\frac{\nu_0}{4}\right|^{\frac{1}{3}} - \left|\nu-\frac{\nu_0}{4}\right|^{\frac{1}{3}}\right|^3,\\
C^+(\nu) &= \frac{1}{12\gamma\sqrt{2\sqrt{3}}}\left|\nu+\frac{\nu_0}{2}\right|\left|\left|\nu+\frac{\nu_0}{4}\right| - \left|\nu+\frac{3\nu_0}{4}\right|\right|,\\
C^0(\nu) &= -C^+(\nu) - C^+(\nu-\nu_0),\\
C^-(\nu) &= C^+(\nu-\nu_0).
\end{aligned} \tag{2.4}$$

However, for $N = a^3$ (a is the scale factor and for orientation reversal symmetric wave function), the structures of these functions are

$$\begin{aligned}
B(\nu) &= \frac{1}{\nu},\\
C^+(\nu) &= \frac{\sqrt{3}}{8\gamma}\left(\nu+\frac{\nu_0}{2}\right),\\
C^0(\nu) &= -\frac{\sqrt{3}}{4\gamma}\nu,\\
C^-(\nu) &= C^+(\nu-\nu_0),
\end{aligned} \tag{2.5}$$

and in the semiclassical limit $\nu \gg \nu_0$, these expressions (2.5) are in agreement with (2.4).

According to the standard definition, the solutions of the first quantized theories correspond to the free field solutions in the second quantized formalism, the solutions of the second quantized theory should corresponds to free field solutions in the third quantized formalism. This means, the solution of loop quantum cosmology must correspond to the classical field of group field cosmology. Now, the free field action for bosonic distribution of universes, of which classical solution reproduces the Hamiltonian constraint for loop quantum gravity, is defined by [28]

$$S_b = \sum_{\nu} \int d\phi \ \mathcal{L}_b = \sum_{\nu} \int d\phi \ \Phi(\nu, \phi) K^2 \Phi(\nu, \phi), \tag{2.6}$$

where $\Phi(\nu, \phi)$ is a real scalar field.

Now, it is worthwhile to analyse the fermionic distribution of universes because it might lead the correct value of the cosmological constant. Since the correct value of cosmological constant has not been obtained through the consideration of only bosonic distributions of universes in the multi-universe [42]. Consequently, the free action corresponding to fermionic group field cosmology is demonstrated by [43]

$$S_f = \sum_{\nu} \int d\phi \ \Psi^b(\nu, \phi) K^a_b \Psi_a(\nu, \phi), \tag{2.7}$$

where $\Psi_a(\nu, \phi) = (\Psi_1(\nu, \phi), \Psi_2(\nu, \phi))$ is a fermionic spinor field and K_{ab} is defined as $K_{ab} = (\gamma^\mu)_{ab} K_\mu$. These spinor indices are raised and lowered by the second-rank antisymmetric tensors C^{ab} and C_{ab}, respectively. The antisymmetric tensors satisfy following condition $C_{ab}C^{cb} = \delta^c_a$ [44].

The above mentioned bosonic and the fermionic actions describe the bosonic and the fermionic universes and hence, it is worthwhile to construct a supersymmetric gauge invariant multiverse. Henceforth, the main idea behind the third quantization is to treat the many-universe system as a quantum field theory on superspace.

III. INTERACTIONS IN SUPERSPACE

In this section we discuss the interaction for the supersymmetric group field cosmology in superspace. For this purpose the superspace propagator's can be used for analyzing superspace perturbations. For instance, the loop diagrams can be studied using the propagator $< 0|\Phi(x,\theta,\bar{\theta})\Phi(x',\theta',\theta')|0 >$ if we consider a cubic interaction term of the form $\mathcal{L}_{int} = \lambda d^2\theta\Phi^3/3$. Now, utilizing Fourier transformation, we are able to write this term as following:

$$\begin{aligned} \mathcal{X}(x,\theta,\bar{\theta},x',\theta',\bar{\theta}') &= \frac{1}{2}\int d^3x \exp i(p_1(x-x')) \\ &\times < 0|\Phi(x,\theta,\bar{\theta})\Phi(x',\theta',\bar{\theta}')\mathcal{B}|0 >, \end{aligned} \tag{3.1}$$

where

$$\mathcal{B} = \frac{\lambda^2}{9}\int d^3x_1 d^2\theta_1 d^3x_2 d^2\theta_2 \Phi^3(x_1,\theta_1,\bar{\theta}_1)\Phi^3(x_2,\theta_2,\bar{\theta}_2). \tag{3.2}$$

However, if we use a cubic interaction term of the form $\mathcal{L}_{int} = \lambda^* d^2\bar{\theta}\bar{\Phi}^3/3$ for propagator for $< 0|\bar{\Phi}(x,\theta,\bar{\theta})\bar{\Phi}(x',\theta',\bar{\theta}')|0 >$ to calculate one loop contributions the similar results hold though we obtain the following expression

$$\begin{aligned} \mathcal{X}(x,\theta,\bar{\theta},x',\theta',\bar{\theta}') &= \frac{1}{2}\int d^3x \exp i(p_1(x-x')) \\ &\times < 0|\Phi(x,\theta,\bar{\theta})\bar{\Phi}(x',\theta',\bar{\theta}')\mathcal{B}|0 >, \end{aligned} \tag{3.3}$$

where

$$\begin{aligned} \mathcal{B} &= 2\lambda\lambda^* \left(\frac{i}{16}\right)^2 \int d^3p_2 d^2\theta_1 d^2\bar{\theta}_2 \\ &\times \frac{\bar{D}_1^2 D_2^2 \delta^2(\theta_1-\theta_2)\delta^2(\bar{\theta}_1-\bar{\theta}_2)}{{p_2}^2 + 2k_{\mu\nu}{p_2}^\mu {p_2}^\mu + k^{\tau\mu}k_{\tau\nu}{p_2}_\mu {p_2}^\nu} \\ &\times \frac{\bar{D}_1^2 D_2^2 \delta^2(\theta_1-\theta_2)\delta^2(\bar{\theta}_1-\bar{\theta}_2)}{\mathcal{A}(p_1-p_2,k)}. \end{aligned} \tag{3.4}$$

It is obvious that the above integral is ambiguous (divergent) and hence required to renormalize the superfield. To justify this a familiar argument for the tadpole graphs is given which vanish for only one external line.

Furthermore, we realize that the super-group field cosmology will have no vertex corrections for both $\mathcal{L}_{int} = \lambda d^2\theta\Phi^3/3$ and $\mathcal{L}_{int} = \lambda^* d^2\bar{\theta}\bar{\Phi}^3/3$ type interactions at one loop. Consequently, these superspace propagators can be used for performing various perturbative calculations. However, for the two loops case there is a diagram with two vertices, and so we define

$$\begin{aligned}
\mathcal{B} &= 4\lambda\lambda^*(\frac{i}{16})^3 \int d^3p_1 d^3p_2 d^2\theta_1 d^2\bar{\theta}_2 \\
&\quad \times \frac{\bar{D}^2_{-p_1-p_2} D^2_{-p_1-p_2} \delta^2(\theta_1-\theta_2)\delta^2(\bar{\theta}_1-\bar{\theta}_2)}{\mathcal{A}(p_1+p_2,k)} \\
&\quad \times \frac{\bar{D}^2_{p_1} D^2_{p_1} \delta^2(\theta_1-\theta_2)\delta^2(\bar{\theta}_1-\bar{\theta}_2)}{{p_1}^2 + 2k_{\mu\nu}{p_1}^\mu {p_1}^\mu + k^{\tau\mu}k_{\tau\nu}{p_1}_\mu {p_1}^\nu} \\
&\quad \times \frac{\bar{D}^2_{p_2} D^2_{p_2} \delta^2(\theta_1-\theta_2)\delta^2(\bar{\theta}_1-\bar{\theta}_2)}{({p_2}^2 + 2k_{\mu\nu}{p_2}^\mu {p_2}^\mu + k^{\tau\mu}k_{\tau\nu}{p_2}_\mu {p_2}^\nu)} \\
&= 4\lambda\lambda^*(\frac{i}{16})^3 \int d^3p_1 d^3p_2 d^2\theta_1 d^2\bar{\theta}_2 \\
&\quad \times \frac{1}{{p_1}^2 + 2k_{\mu\nu}{p_1}^\mu {p_1}^\mu + k^{\tau\mu}k_{\tau\nu}{p_1}_\mu {p_1}^\nu} \\
&\quad \times \frac{1}{{p_2}^2 + 2k_{\mu\nu}{p_2}^\mu {p_2}^\mu + k^{\tau\mu}k_{\tau\nu}{p_2}_\mu {p_2}^\nu} \\
&\quad \times \frac{1}{\mathcal{A}(p_1+p_2,k)},
\end{aligned} \tag{3.5}$$

which also vanishes even at two-loops. This observation provides us a feeling that the vacuum energy for superspace still vanishes at two-loops. Therefore, it would be interesting to analyse general non-renormalization theorems in this superspace.

IV. CONCLUSION

In this paper, we have analysed the group field cosmology with $\mathcal{N}=2$ supersymmetry in $\mathcal{N}=2$ superspace. We have analysed the group field cosmology in a representation where a non-trivial mixing between the original generators of $\mathcal{N}=2$ supersymmetry occurred. We have obtained an explicit expression for supercharges and superderivatives in this representation of $\mathcal{N}=2$ supersymmetry. We have used these superderivatives in superspace to derive explicit expressions for propagators of the group field cosmology model. Furthermore, we have found that these propagators play an important role in perturbative calculations. We thus have observed that the vacuum energy for such theory is zero even at two-loops which leads an interesting result. We hope that the present investigation will play a crucial role in building a systematic theory for multiverse.

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