

On Molecular descriptors with python of 3D Lattice

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Abstract—Topological index of a graph G is a numerical value which reveals it's topological properties. A bravais lattice is an infinite arrangement of discrete molecules with orientation and arrangement appears exactly same. We give properties for the connectivity and neighbourhood degree based indices for frequently used three dimensional type lattice .

Keywords—*bounds, connectivity, lattice, neighborhood, topological indices.*

MSC (2010): 05C12.

I. INTRODUCTION

When the graph (G) represents the molecular structure of a chemical compound, it is called a molecular graph. This types of graphs are widely used in designing and create new things in all field of Science. First topological index was introduced by Wiener. We present various neighborhood degree based index of widely used chemical structures which will appear in mathematical chemistry and solid state physics. Solid state physics is study about solid through crystallography, metallurgy and others.

Molecular structure modeling is High priority in Proteins , Enzymes , Medical drugs, Virus, Bacteria, Field of Nano Science etc. There are so many 2D and 3D models to investigate the property and the activity of molecules. Simple 3D molecular formula is CH_4 .

Lattices introduced by August Bravais (1811-1863). Bravais lattices include cubic , face centered cubic and body centered cubic. Another one is hexagonal close packed arrangements. In adding up to translational symmetry the unit cell must have some symmetry to arrangement of the atoms with in the unit cell.

The precise combination of evenness elements in a crystal defines its space group which are 230 different space groups. Another main concept in lattices is reciprocal lattice. X-ray Crystallographers use a reciprocal lattice defined by 3 vectors. Just like real space lattice reciprocal lattice constructed from a series of primitive cells(Wigner Seitz cells) which entirely fill the space without overlapping and without gaps. Reciprocal Lattice's Wigner Seitz cell is called as first Brillouin zone in electronic system.

Lattices investigated from eighteenth century by many mathematicians Gauss, Lagrange later Minkowski . Lattice points arranged like, each has one and the same background. To fill the space in efficient manner to maintain the surroundings. Its points denote the number of atoms in our basis. Recently it is active topic in research in the fields of Computer Science , Chemistry, Physics and Cryptography. Here we are found various descriptors for one type of face centered Lattice.

1.1 Definitions

Various indices were introduced in various periods of time,

Vertex level based topological index was introduced by M. Randic [9] in 1975, Connectivity index or Randic index.

$$R(A) = \sum_{uv \in E(A)} \left(\frac{1}{\sqrt{\deg(u) \deg(v)}} \right)$$

The inverse Randic index [10] is

$$RR(A) = \sum_{uv \in E(A)} \sqrt{\deg(u) \deg(v)}$$

B. Zhou and N. Trinanjstic - sum connectivity index [13] given by

$$SCI(A) = \sum_{uv \in E(A)} \left(\frac{1}{\sqrt{\deg(u) + \deg(v)}} \right)$$

SDD index [14] is

$$\text{SDD}(A) = \sum_{uv \in E(A)} \left[\frac{\deg(u)}{\deg(v)} + \frac{\deg(v)}{\deg(u)} \right]$$

The 1st and 2nd Zagreb indices presented by Gutman and Trinajstić in [13,14].

$$M_1(A) = \sum_{uv \in E(A)} (\deg(u) + \deg(v))$$

$$M_2(A) = \sum_{uv \in E(A)} (\deg(u) \deg(v))$$

Furtula and Gutman [15] presented the forgotten topological index as:

$$F(A) = \sum_{v \in V(A)} \deg(v)^3 = \sum_{uv \in E(A)} (\deg(u)^2 + \deg(v)^2)$$

Ghorbani and Azimi [12] defined the two multiple Zagreb indices of a graph A are

$$\text{PM}_1(A) = \prod_{uv \in E(A)} (\deg(u) + \deg(v))$$

$$\text{PM}_2(A) = \prod_{uv \in E(A)} \deg(u) \deg(v)$$

Urtula [17] et al. have given an augmented Zagreb index as

$$\text{AZI}(A) = \sum_{uv \in E(A)} \left(\frac{\deg(u) \deg(v)}{\deg(u) + \deg(v) - 2} \right)^3$$

The Balaban index [18,19]

$$J(A) = \frac{m}{m-n+2} \sum_{uv \in E(A)} \left(\frac{1}{\sqrt{\deg(u) \deg(v)}} \right)$$

The new versions of the Zagreb indices introduced by Ranjini et al. [20]

$$\text{ReZ}_1(A) = \sum_{uv \in E(A)} \frac{\deg(u) + \deg(v)}{\deg(u) \deg(v)}$$

$$\text{ReZ}_2(A) = \sum_{uv \in E(A)} \frac{\deg(u) \deg(v)}{\deg(u) + \deg(v)}$$

$$\text{ReZ}_3(A) = \sum_{uv \in E(A)} \deg(u) \deg(v) (\deg(u) + \deg(v))$$

Fath-Tabar [21] introduced two Zagreb polynomials of a graph A

$$M_1(A,x) = \sum_{uv \in E(A)} x^{(\deg(u)+\deg(v))}$$

$$M_2(A,x) = \sum_{uv \in E(A)} x^{(\deg(u)\deg(v))}$$

Chaluvaraju et al. [22] redefined the two hyper-Zagreb polynomials as

$$HM_1(A,x) = \sum_{uv \in E(A)} x^{(\deg(u)+\deg(v))^2}$$

$$HM_2(A,x) = \sum_{uv \in E(A)} x^{(\deg(u)\deg(v))^2}$$

the geometric arithmetic index GA , introduced in [28]

$$GA_1(A) = \sum_{uv \in E(A)} \frac{2\sqrt{\deg(u)\deg(v)}}{\deg(u) + \deg(v)}$$

The atom–bond connectivity index ABC

$$ABC(A) = \sum_{uv \in E(A)} \sqrt{\frac{\deg(u) + \deg(v) - 2}{\deg(u)\deg(v)}}$$

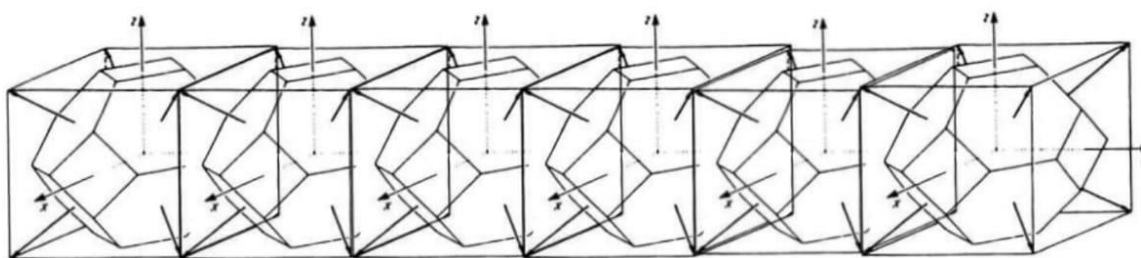
2. Face centered cubic Lattice

Face centered cubic lattice has gathering of cubes with atom at faces of each cube and also atoms at the edges. Solid state material have electronic, Physical and chemical properties. Wide variety of descriptors have developed to study the characteristic and properties through their structure. A crystalline system can be made by staking replica of some repeating unit in an organized manner in without break and without overlap. The structure specified by defining the shape and size of the cell and location of atoms within it. There are 14 distinct type basic unit cell these are bravais lattices. Materials with this structure include gold, aluminum, nickel silver, and copper. Lattice have some special properties of particular value for solid state electronic structure calculations.

Methodology for calculations:

The structure of graph A contains $20n+4$ vertices and $32n+4$ edges. To find the molecular descriptors we subdivide the Edge set into 3 partitions. The First edge set contains $4(n-1)$ edges where $\deg(u)=4$ and $\deg(v)=4$. The second edge set contains $4(n-1)$ edges where $\deg(u)=3$ and $\deg(v)=4$. The third edge set contains $24n+12$ edges where $\deg(u)=3$ and $\deg(v)=3$. Table 1 shows the frequency as follows

<i>Degree</i>	<i>frequency</i>	<i>Edge set</i>
(4,4)	$4(n-1)$	E_1
(4,3)	$4(n-1)$	E_2
(3,3)	$24n+12$	E_3



2.1 THEOREM

LET A BE AN BRAVAIS LATTICE, FOR ALL $N \geq 2$. THEN THE RANDIC INDEX IS

$$R(A) = (n - 1) + \frac{4(n - 1)}{\sqrt{12}} + \frac{24n + 12}{3}$$

Proof:

Let A be an bravais Lattice. Then

$$R(A) = \sum_{uv \in E(A)} \left(\frac{1}{\sqrt{\deg(u) \deg(v)}} \right)$$

$$R(A) = \sum_{uv \in E_1(A)} \left(\frac{1}{\sqrt{\deg(u) \deg(v)}} \right) + \sum_{uv \in E_2(A)} \left(\frac{1}{\sqrt{\deg(u) \deg(v)}} \right) + \sum_{uv \in E_3(A)} \left(\frac{1}{\sqrt{\deg(u) \deg(v)}} \right)$$

$$R(A) = (n-1) + \frac{4(n-1)}{\sqrt{12}} + \frac{24n+12}{3}$$

2.2 THEOREM

Let A be an bravais Lattice, for all $n \geq 2$. Then the inverse Randic index is

$$RR(A) = 72n + 4\sqrt{12}n + 20 - 4\sqrt{12}$$

Proof:

Let A be an bravais Lattice. Then

$$RR(A) = \sum_{uv \in E(A)} \sqrt{\deg(u) \deg(v)}$$

$$RR(A) = \sum_{uv \in E_1(A)} \sqrt{\deg(u) \deg(v)} + \sum_{uv \in E_2(A)} \sqrt{\deg(u) \deg(v)} + \sum_{uv \in E_3(A)} \sqrt{\deg(u) \deg(v)}$$

$$RR(A) = 4(n-1)\sqrt{16} + 4(n-1)\sqrt{12} + (24n+12)\sqrt{9}$$

$$RR(A) = 72n + 4\sqrt{12}n + 20 - 4\sqrt{12}$$

2.3 THEOREM

Let A be an bravais Lattice, for all $n \geq 2$. Then sum connectivity index is

$$SCI(A) = \frac{4(n-1)}{\sqrt{8}} + \frac{4(n-1)}{\sqrt{7}} + \frac{24n+12}{\sqrt{6}}$$

Proof:

Let A be an bravais Lattice. Then

$$SCI(A) = \sum_{uv \in E(A)} \left(\frac{1}{\sqrt{\deg(u) + \deg(v)}} \right)$$

$$SCI(A) = \sum_{uv \in E_1(A)} \left(\frac{1}{\sqrt{\deg(u) + \deg(v)}} \right) + \sum_{uv \in E_2(A)} \left(\frac{1}{\sqrt{\deg(u) + \deg(v)}} \right) + \sum_{uv \in E_3(A)} \left(\frac{1}{\sqrt{\deg(u) + \deg(v)}} \right)$$

$$SCI(A) = \frac{4(n-1)}{\sqrt{8}} + \frac{4(n-1)}{\sqrt{7}} + \frac{24n+12}{\sqrt{6}}$$

2.4 THEOREM

Let A be an bravais Lattice, for all $n \geq 2$. Then SSD index is

$$SSD(A) = \frac{193}{3}n + \frac{23}{3}$$

Proof:

Let A be an bravais Lattice. Then the SDD index is

$$SDD(A) = \sum_{uv \in E(A)} \left[\frac{\deg(u)}{\deg(v)} + \frac{\deg(v)}{\deg(u)} \right]$$

$$SDD(A) = \sum_{uv \in E_1(A)} \left[\frac{\deg(u)}{\deg(v)} + \frac{\deg(v)}{\deg(u)} \right] + \sum_{uv \in E_2(A)} \left[\frac{\deg(u)}{\deg(v)} + \frac{\deg(v)}{\deg(u)} \right] + \sum_{uv \in E_3(A)} \left[\frac{\deg(u)}{\deg(v)} + \frac{\deg(v)}{\deg(u)} \right]$$

$$SDD(A) = \left(\frac{4}{4} + \frac{4}{4} \right) 4(n-1) + \left(\frac{4}{3} + \frac{3}{4} \right) 4(n-1) + \left(\frac{3}{3} + \frac{3}{3} \right) (24n+12)$$

$$SSD(A) = \frac{193}{3}n + \frac{23}{3}$$

2.5 THEOREM

Let A be an bravais Lattice, for all $n \geq 2$. Then Zagreb indices are

$$M_1(A) = 244n - 64$$

$$M_2(A) = 328n - 6$$

Proof:

Let A be an bravais Lattice. Then

$$M_1(A) = \sum_{uv \in E(A)} (\deg(u) + \deg(v))$$

$$M_1(A) = \sum_{uv \in E_1(A)} (\deg(u) + \deg(v)) + \sum_{uv \in E_2(A)} (\deg(u) + \deg(v)) + \sum_{uv \in E_3(A)} (\deg(u) + \deg(v))$$

$$M_1(A) = 424 + (n-2)244$$

$$M_1(A) = 244n - 64$$

$$M_2(A) = \sum_{uv \in E(A)} (\deg(u) \deg(v))$$

$$M_2(A) = 652 + (n-2)328$$

$$M_2(A) = 328n - 6$$

2.6 THEOREM

Let A be an bravais Lattice, for all $n \geq 2$. Then the Forgotton index is

$$F(A) = 796n - 256$$

Proof:

Let A be an bravais Lattice. Then

$$F(A) = \sum_{v \in V(A)} \deg(v)^3 = \sum_{uv \in E(A)} (\deg(u)^2 + \deg(v)^2)$$

$$F(A) = 1336 + (n-2)796$$

$$F(A) = 796n - 256$$

2.7 THEOREM

Let A be an bravais Lattice, for all $n \geq 2$. Then the multiple Zagreb indices are

$$PM_1(A) = 8^{4(n-1)} \times 7^{4(n-1)} \times 6^{24n+12}$$

$$PM_2(A) = 16^{4(n-1)} \times 12^{4(n-1)} \times 9^{24n+12}$$

Proof:

Consider a bravais Lattice A. Then

$$PM_1(A) = \prod_{uv \in E(A)} \deg(u) + \deg(v)$$

$$PM_1(A) = \prod_{uv \in E_1(A)} \deg(u) + \deg(v) \times \prod_{uv \in E_2(A)} \deg(u) + \deg(v) \times \prod_{uv \in E_3(A)} \deg(u) + \deg(v)$$

$$PM_1(A) = 8^{4(n-1)} \times 7^{4(n-1)} \times 6^{24n+12}$$

$$PM_2(A) = \prod_{uv \in E_1(A)} \deg(u) \deg(v) \times \prod_{uv \in E_2(A)} \deg(u) \deg(v) \times \prod_{uv \in E_3(A)} \deg(u) \deg(v)$$

$$PM_2(A) = 16^{4(n-1)} \times 12^{4(n-1)} \times 9^{24n+12}$$

2.8 THEOREM

Let A be an bravais Lattice, for all $n \geq 2$. Then the augmented Zagreb index is

$$AZI(A) = 4 \left(\frac{16}{6} \right)^3 (n-1) + 4 \left(\frac{12}{5} \right)^3 (n-1) + \left(\frac{9}{4} \right)^3 (24n+12)$$

Proof:

Consider a bravais Lattice A. Then

$$AZI(A) = \sum_{uv \in E(A)} \left(\frac{\deg(u)\deg(v)}{\deg(u) + \deg(v) - 2} \right)^3$$

$$AZI(A) =$$

$$\sum_{uv \in E_1(A)} \left(\frac{\deg(u)\deg(v)}{\deg(u) + \deg(v) - 2} \right)^3 + \sum_{uv \in E_2(A)} \left(\frac{\deg(u)\deg(v)}{\deg(u) + \deg(v) - 2} \right)^3 + \sum_{uv \in E_3(A)} \left(\frac{\deg(u)\deg(v)}{\deg(u) + \deg(v) - 2} \right)^3$$

$$AZI(A) = 4 \left(\frac{16}{6} \right)^3 (n-1) + 4 \left(\frac{12}{5} \right)^3 (n-1) + \left(\frac{9}{4} \right)^3 (24n+12)$$

2.9 THEOREM

Let A be an bravais Lattice, for all $n \geq 2$. Then the redefined Zagreb indices are

$$ReZ_1(A) = 2(n-1) + \frac{7(n-1)}{3} + \frac{2}{3}(24n+12)$$

$$ReZ_2(A) = 8(n-1) + \frac{48(n-1)}{7} + \frac{3}{2}(24n+12)$$

$$ReZ_3(A) = 512(n-1) + 336(n-1) + 54(24n+12)$$

Proof:

Consider a bravais Lattice A. Then the redefined Zagreb indices are as follows

$$ReZ_1(A) = \sum_{uv \in E(A)} \frac{\deg(u) + \deg(v)}{\deg(u)\deg(v)}$$

$$ReZ_1(A) = \sum_{uv \in E_1(A)} \frac{\deg(u) + \deg(v)}{\deg(u)\deg(v)} + \sum_{uv \in E_2(A)} \frac{\deg(u) + \deg(v)}{\deg(u)\deg(v)} + \sum_{uv \in E_3(A)} \frac{\deg(u) + \deg(v)}{\deg(u)\deg(v)}$$

$$\text{ReZ}_1(A) = 2(n-1) + \frac{7(n-1)}{3} + \frac{2}{3}(24n+12)$$

$$\text{ReZ}_2(A) = \sum_{uv \in E_1(A)} \frac{\text{deg}(u)\text{deg}(v)}{\text{deg}(u) + \text{deg}(v)} + \sum_{uv \in E_2(A)} \frac{\text{deg}(u)\text{deg}(v)}{\text{deg}(u) + \text{deg}(v)} + \sum_{uv \in E_3(A)} \frac{\text{deg}(u)\text{deg}(v)}{\text{deg}(u) + \text{deg}(v)}$$

$$\text{ReZ}_2(A) = 8(n-1) + \frac{48(n-1)}{7} + \frac{3}{2}(24n+12)$$

$$\text{ReZ}_3(A) = \sum_{uv \in E(A)} \text{deg}(u)\text{deg}(v)(\text{deg}(u) + \text{deg}(v))$$

$$\text{ReZ}_3(A) = \sum_{uv \in E_1(A)} \text{deg}(u)\text{deg}(v)(\text{deg}(u) + \text{deg}(v)) + \sum_{uv \in E_2(A)} \text{deg}(u)\text{deg}(v)(\text{deg}(u) + \text{deg}(v)) + \sum_{uv \in E_3(A)} \text{deg}(u)\text{deg}(v)(\text{deg}(u) + \text{deg}(v))$$

$$\text{ReZ}_3(A) = 512(n-1) + 336(n-1) + 54(24n+12)$$

2.10. THEOREM

Let A be an bravais Lattice, for all $n \geq 2$. Then the Balaban index is

$$J(A) = \frac{32n+4}{12n+2} \left[4(n-1) \left(\frac{1}{\sqrt{16}} \right) + 4(n-1) \left(\frac{1}{\sqrt{12}} \right) + 24n+12 \left(\frac{1}{\sqrt{9}} \right) \right]$$

Proof:

Consider a bravais Lattice A. Then

$$J(A) = \frac{m}{m-n+2} \sum_{uv \in E(A)} \left(\frac{1}{\sqrt{\text{deg}(u)\text{deg}(v)}} \right)$$

$$J(A) = \frac{m}{m-n+2} \left(\sum_{uv \in E_1(A)} \left(\frac{1}{\sqrt{\text{deg}(u)\text{deg}(v)}} \right) + \sum_{uv \in E_2(A)} \left(\frac{1}{\sqrt{\text{deg}(u)\text{deg}(v)}} \right) + \sum_{uv \in E_3(A)} \left(\frac{1}{\sqrt{\text{deg}(u)\text{deg}(v)}} \right) \right)$$

$$J(A) = \frac{32n+4}{12n+2} \left[4(n-1) \left(\frac{1}{\sqrt{16}} \right) + 4(n-1) \left(\frac{1}{\sqrt{12}} \right) + 24n+12 \left(\frac{1}{\sqrt{9}} \right) \right]$$

2.11 THEOREM

Let A be an bravais Lattice, for all $n \geq 2$. Then the geometric arithmetic index is

$$GA_1(A) = \frac{8\sqrt{12}}{7}(n-1) + 28n + 8$$

Proof:

Consider a bravais Lattice A. Then

$$GA_1(A) = \sum_{uv \in E(A)} \frac{2\sqrt{\deg(u)\deg(v)}}{\deg(u) + \deg(v)}$$

$$GA_1(A) = \frac{8(n-1)\sqrt{16}}{8} + \frac{8(n-1)\sqrt{12}}{7} + (24+12)\frac{2\sqrt{9}}{6}$$

$$GA_1(A) = \frac{8\sqrt{12}}{7}(n-1) + 28n + 8$$

2.12 THEOREM

Let A be an bravais Lattice, for all $n \geq 2$. Then the atom-bond connectivity index is

$$ABC(A) = (n-1)4\sqrt{\frac{6}{16}} + 4(n-1)\sqrt{\frac{5}{12}} + (24n+12)\sqrt{\frac{4}{9}}$$

Proof:

Consider a bravais Lattice A. Then atom-bond connectivity index is

$$ABC(A) = \sum_{uv \in E(A)} \sqrt{\frac{\deg(u) + \deg(v) - 2}{\deg(u)\deg(v)}}$$

$$ABC(A) = \sum_{uv \in E_1(A)} \sqrt{\frac{\deg(u) + \deg(v) - 2}{\deg(u)\deg(v)}} + \sum_{uv \in E_2(A)} \sqrt{\frac{\deg(u) + \deg(v) - 2}{\deg(u)\deg(v)}} + \sum_{uv \in E_3(A)} \sqrt{\frac{\deg(u) + \deg(v) - 2}{\deg(u)\deg(v)}}$$

$$ABC(A) = (n-1)4\sqrt{\frac{6}{16}} + 4(n-1)\sqrt{\frac{5}{12}} + (24n+12)\sqrt{\frac{4}{9}}$$

2.13 THEOREM

Let A be an bravais Lattice, for all $n \geq 2$. Then the two Zagreb polynomials are

$$M_1(A,x) = 4(n-1)x^8 + 4(n-1)x^7 + (24n+12)x^6$$

$$M_2(A,x) = 4(n-1)x^{16} + 4(n-1)x^{12} + (24n+12)x^9$$

Proof:

Consider a bravais Lattice A. Then the two Zagreb polynomials are

$$M_1(A,x) = \sum_{uv \in E(A)} x^{(\deg(u)+\deg(v))}$$

$$M_1(A,x) = \sum_{uv \in E_1(A)} x^{(\deg(u)+\deg(v))} + \sum_{uv \in E_2(A)} x^{(\deg(u)+\deg(v))} + \sum_{uv \in E_3(A)} x^{(\deg(u)+\deg(v))}$$

$$M_1(A,x) = 4(n-1)x^8 + 4(n-1)x^7 + (24n+12)x^6$$

$$M_2(A,x) = \sum_{uv \in E(A)} x^{(\deg(u)\deg(v))}$$

$$M_2(A,x) = \sum_{uv \in E_1(A)} x^{(\deg(u)\deg(v))} + \sum_{uv \in E_2(A)} x^{(\deg(u)\deg(v))} + \sum_{uv \in E_3(A)} x^{(\deg(u)\deg(v))}$$

$$M_2(A,x) = 4(n-1)x^{16} + 4(n-1)x^{12} + (24n+12)x^9$$

2.14 THEOREM

Let A be an bravais Lattice, for all $n \geq 2$. Then the two hyper Zagreb polynomials are

$$HM_1(A,x) = 4(n-1)x^{64} + 4(n-1)x^{49} + (24n+12)x^{36}$$

$$HM_2(A,x) = 4(n-1)x^{156} + 4(n-1)x^{144} + (24n+12)x^{82}$$

Proof:

Consider a bravais Lattice A. Then the two hyper Zagreb polynomials are

$$HM_1(A,x) = \sum_{uv \in E(A)} x^{(\deg(u)+\deg(v))^2}$$

$$HM_1(A,x) = \sum_{uv \in E_1(A)} x^{(\deg(u)+\deg(v))^2} + \sum_{uv \in E_2(A)} x^{(\deg(u)+\deg(v))^2} + \sum_{uv \in E_3(A)} x^{(\deg(u)+\deg(v))^2}$$

$$HM_1(A,x) = 4(n-1)x^{64} + 4(n-1)x^{49} + (24n+12)x^{36}$$

$$HM_2(A,x) = \sum_{uv \in E_1(A)} x^{(\deg(u)\deg(v))^2} + \sum_{uv \in E_2(A)} x^{(\deg(u)\deg(v))^2} + \sum_{uv \in E_3(A)} x^{(\deg(u)\deg(v))^2}$$

$$HM_2(A,x) = 4(n-1)x^{156} + 4(n-1)x^{144} + (24n+12)x^{82}$$

3. NEIGHBOURHOOD DEGREESUM BASED INDICES

Sourav Mondal, Nilanjan De, Anita pal introduced neighbourhood degree based indices to check the chemical applicability of indices.

Here we compute and given the relation between other topological indices of neighbourhood degree based indices. The indices are

$$ND_1(A) = \sum_{uv \in E(A)} \sqrt{\delta_A(u)\delta_A(v)}$$

$$ND_2(A) = \sum_{uv \in E(A)} \left(\frac{1}{\sqrt{\delta_A(u) + \delta_A(v)}} \right)$$

$$ND_3(A) = \sum_{uv \in E(A)} \delta_A(u)\delta_A(v)(\delta_A(u) + \delta_A(v))$$

$$ND_4(A) = \sum_{uv \in E(A)} \left(\frac{1}{\sqrt{\delta_A(u)\delta_A(v)}} \right)$$

$$ND_5(A) = \sum_{uv \in E(A)} \left[\frac{\delta_A(u)}{\delta_A(v)} + \frac{\delta_A(v)}{\delta_A(u)} \right]$$

$$ND_6(A) = \sum_{uv \in E(A)} \delta_A(u) \deg(u) + \deg(v) + \delta_A(v)$$

$$\text{Where } \delta_A(u) = \sum_{v \in N_A(u)} \deg(v)$$

The fourth version of atom–bond connectivity index ABC_4 by Ghorbani et al. [23]

$$ABC_4 = \sum_{uv \in E(A)} \sqrt{\frac{\delta_A(u) + \delta_A(v) - 2}{\delta_A(u)\delta_A(v)}}$$

$$\text{Where } \delta_A(u) = \sum_{v \in N_A(u)} \deg(v)$$

The fifth version of the geometric arithmetic index GA_5 by Graovac et al. [24]

$$GA_5(A) = \sum_{uv \in E(A)} \frac{2\sqrt{\delta_A(u)\delta_A(v)}}{\delta_A(u) + \delta_A(v)}$$

Where $\delta_A(u) = \sum_{v \in N_A(u)} \text{deg}(v)$

3.1 . THEOREM

Let A be an bravais Lattice, for all $n \geq 2$. Then the neighbourhood degree based indices

$$ND_1(A) = (32 + 4n)\sqrt{82} + 16(n - 1)\sqrt{90} + 4(n - 1)\sqrt{196} + 8(n - 1)\sqrt{110}$$

$$ND_2(A) = \frac{32 + 4n}{\sqrt{18}} + \frac{16(n - 1)}{\sqrt{19}} + \frac{4(n - 1)}{\sqrt{28}} + \frac{8(n - 1)}{\sqrt{21}}$$

$$ND_3(A) = (32 + 4n)(1458) + 16(n - 1)1710 + 21952(n - 1) + 18480(n - 1)$$

$$ND_4(A) = \frac{32 + 4n}{\sqrt{82}} + \frac{16(n - 1)}{\sqrt{90}} + \frac{4(n - 1)}{\sqrt{196}} + \frac{8(n - 1)}{\sqrt{110}}$$

$$ND_5(A) = 2(32 + 4n) + \left(\frac{9}{10} + \frac{10}{9}\right)16(n - 1) + 8(n - 1) + \left(\frac{10}{11} + \frac{11}{10}\right)8(n - 1)$$

$$ABC_4 = \sqrt{\frac{16}{82}} \times (32 + 4n) + 16(n - 1)\sqrt{\frac{17}{90}} + 4(n - 1)\sqrt{\frac{26}{196}} + 8(n - 1)\sqrt{\frac{19}{110}}$$

$$GA_5(A) = \frac{(32 + 4n)\sqrt{81}}{9} + \frac{32(n - 1)\sqrt{90}}{19} + \frac{2(n - 1)\sqrt{196}}{7} + \frac{16(n - 1)\sqrt{110}}{21}$$

Proof:

$$ND_1(A) = \sum_{uv \in E(A)} \sqrt{\delta_A(u)\delta_A(v)}$$

$$ND_1(A) = \sum_{uv \in E_1(A)} \sqrt{\delta_A(u)\delta_A(v)} + \sum_{uv \in E_2(A)} \sqrt{\delta_A(u)\delta_A(v)} + \sum_{uv \in E_3(A)} \sqrt{\delta_A(u)\delta_A(v)} + \sum_{uv \in E_4(A)} \sqrt{\delta_A(u)\delta_A(v)}$$

$$ND_1(A) = (32 + 4n)\sqrt{82} + 16(n - 1)\sqrt{90} + 4(n - 1)\sqrt{196} + 8(n - 1)\sqrt{110}$$

$$ND_2(A) = \sum_{uv \in E(A)} \left(\frac{1}{\sqrt{\delta_A(u) + \delta_A(v)}} \right)$$

$$ND_2(A) = \sum_{uv \in E_1(A)} \left(\frac{1}{\sqrt{\delta_A(u) + \delta_A(v)}} \right) + \sum_{uv \in E_2(A)} \left(\frac{1}{\sqrt{\delta_A(u) + \delta_A(v)}} \right) + \sum_{uv \in E_3(A)} \left(\frac{1}{\sqrt{\delta_A(u) + \delta_A(v)}} \right) + \sum_{uv \in E_4(A)} \left(\frac{1}{\sqrt{\delta_A(u) + \delta_A(v)}} \right)$$

$$ND_2(A) = \frac{32 + 4n}{\sqrt{18}} + \frac{16(n-1)}{\sqrt{19}} + \frac{4(n-1)}{\sqrt{28}} + \frac{8(n-1)}{\sqrt{21}}$$

$$ND_3(A) = \sum_{uv \in E_1(A)} \delta_A(u) \delta_A(v) (\delta_A(u) + \delta_A(v)) + \sum_{uv \in E_2(A)} \delta_A(u) \delta_A(v) (\delta_A(u) + \delta_A(v)) \\ + \sum_{uv \in E_3(A)} \delta_A(u) \delta_A(v) (\delta_A(u) + \delta_A(v)) + \sum_{uv \in E_4(A)} \delta_A(u) \delta_A(v) (\delta_A(u) + \delta_A(v))$$

$$ND_3(A) = (32 + 4n)(81 \times 18) + 16(n-1)(90 \times 19) + 21952(n-1) + 18480(n-1)$$

$$ND_4(A) = \sum_{uv \in E(A)} \left(\frac{1}{\sqrt{\delta_A(u) \delta_A(v)}} \right)$$

$$ND_4(A) =$$

$$\sum_{uv \in E_1(A)} \left(\frac{1}{\sqrt{\delta_A(u) \delta_A(v)}} \right) + \sum_{uv \in E_2(A)} \left(\frac{1}{\sqrt{\delta_A(u) \delta_A(v)}} \right) + \sum_{uv \in E_3(A)} \left(\frac{1}{\sqrt{\delta_A(u) \delta_A(v)}} \right) + \sum_{uv \in E_4(A)} \left(\frac{1}{\sqrt{\delta_A(u) \delta_A(v)}} \right)$$

$$ND_4(A) = \frac{32 + 4n}{\sqrt{82}} + \frac{16(n-1)}{\sqrt{90}} + \frac{4(n-1)}{\sqrt{196}} + \frac{8(n-1)}{\sqrt{110}}$$

$$ND_5(A) = \sum_{uv \in E(A)} \left[\frac{\delta_A(u)}{\delta_A(v)} + \frac{\delta_A(v)}{\delta_A(u)} \right]$$

$$ND_5(A) = \sum_{uv \in E_1(A)} \left[\frac{\delta_A(u)}{\delta_A(v)} + \frac{\delta_A(v)}{\delta_A(u)} \right] + \sum_{uv \in E_2(A)} \left[\frac{\delta_A(u)}{\delta_A(v)} + \frac{\delta_A(v)}{\delta_A(u)} \right] + \sum_{uv \in E_3(A)} \left[\frac{\delta_A(u)}{\delta_A(v)} + \frac{\delta_A(v)}{\delta_A(u)} \right] +$$

$$\sum_{uv \in E_4(A)} \left[\frac{\delta_A(u)}{\delta_A(v)} + \frac{\delta_A(v)}{\delta_A(u)} \right]$$

$$ND_5(A) = 2(32 + 4n) + \left(\frac{9}{10} + \frac{10}{9} \right) 16(n-1) + 8(n-1) + \left(\frac{10}{11} + \frac{11}{10} \right) 8(n-1)$$

$$ND_6(A) = \sum_{uv \in E(A)} \delta_A(u) \deg(u) + \deg(v) + \delta_A(v)$$

The fourth version of atom-bond connectivity index ABC_4 by Ghorbani et al. [23]

$$ABC_4 = \sum_{uv \in E(A)} \sqrt{\frac{\delta_A(u) + \delta_A(v) - 2}{\delta_A(u) \delta_A(v)}}$$

Where $\delta_A(u) = \sum_{v \in N_A(u)} \text{deg}(v)$

ABC₄=

$$\sum_{uv \in E_1(A)} \sqrt{\frac{\delta_A(u) + \delta_A(v) - 2}{\delta_A(u)\delta_A(v)}} + \sum_{uv \in E_2(A)} \sqrt{\frac{\delta_A(u) + \delta_A(v) - 2}{\delta_A(u)\delta_A(v)}} + \sum_{uv \in E_3(A)} \sqrt{\frac{\delta_A(u) + \delta_A(v) - 2}{\delta_A(u)\delta_A(v)}} + \sum_{uv \in E_4(A)} \sqrt{\frac{\delta_A(u) + \delta_A(v) - 2}{\delta_A(u)\delta_A(v)}}$$

$$ABC_4 = \sqrt{\frac{16}{82}} \times (32 + 4n) + 16(n - 1)\sqrt{\frac{17}{90}} + 4(n - 1)\sqrt{\frac{26}{196}} + 8(n - 1)\sqrt{\frac{19}{110}}$$

Where $\delta_A(u) = \sum_{v \in N_A(u)} \text{deg}(v)$

The fifth version of the geometric arithmetic index GA₅ by Graovac et al. [24]

$$GA_5(A) = \sum_{uv \in E(A)} \frac{2\sqrt{\delta_A(u)\delta_A(v)}}{\delta_A(u) + \delta_A(v)}$$

Where $\delta_A(u) = \sum_{v \in N_A(u)} \text{deg}(v)$

GA₅(A)=

$$\sum_{uv \in E_1(A)} \frac{2\sqrt{\delta_A(u)\delta_A(v)}}{\delta_A(u) + \delta_A(v)} + \sum_{uv \in E_2(A)} \frac{2\sqrt{\delta_A(u)\delta_A(v)}}{\delta_A(u) + \delta_A(v)} + \sum_{uv \in E_3(A)} \frac{2\sqrt{\delta_A(u)\delta_A(v)}}{\delta_A(u) + \delta_A(v)} + \sum_{uv \in E_4(A)} \frac{2\sqrt{\delta_A(u)\delta_A(v)}}{\delta_A(u) + \delta_A(v)}$$

$$GA_5(A) = \frac{(32 + 4n)\sqrt{81}}{9} + \frac{32(n - 1)\sqrt{90}}{19} + \frac{2(n - 1)\sqrt{196}}{7} + \frac{16(n - 1)\sqrt{110}}{21}$$

Example Python Program for SSD index calculation of given lattice

```
n=int(input("Enter n value :"))
```

```
R=(193/3)*n+(23/3)
```

```
print(R)
```

Likewise we can use for trouble-free calculation.

Conclusion:

We find Topological indices namely Zagreb index, arithmetic geometric index, multiple Zagreb

index, redefined Zagreb index for finding the total π - electron energy of molecules of one type of Bravais Lattice using python program. More reference readers refer <https://repl.it/@Manimekalai/eccentricity-based-index>

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