# Effect of Subgrade Reaction on Displacement and Bending Moment of a Pile Using Spectral Response Analysis

**[1]Tejaswini, L [2]Darga Kumar, N**

[1]Post Graduate Student and [2]Associate Professor in Civil Engineering

JNTUH University College of Engineering, Science and Technology, Hyderabad, India

[1]lingamtejaswini.dce@gmail.com

[2]ndkumar@jntuh.ac.in

**Abstract:**

In this study the influence of horizontal subgrade reaction on the dynamic response of single pile under seismic loading using spectral response analysis as per IS 1893 is presented. Free-head and fixed-head piles with diameters of 0.3–0.5 m and lengths of 12–18 m were analysed in sandy soils with depth-dependent horizontal subgrade modulus values of 1.0, 1.5, and 2.0 kg/cm³. The analysis considered spectral displacement, bending moment, first natural period, first natural circular frequency and soil reaction. Results showed that the pile diameter governs the seismic behaviour compared to the pile length. The larger diameter piles resulted lower displacement, but sustaining more bending moment. Beyond 14 m embedment depth of pile, the added pile length offers minimal benefits in terms of soil reaction, bending resistance. Free-head piles showed higher displacements (up to ~19.7 mm for 0.3 m diameter piles at $\eta_h$=2.0 kg/cm³) but lower bending moments, while fixed-head piles showed less displacements, but bending moments are exceeding 1500 kN-m. The findings provide design insights for bridges, high-rise buildings, metro systems, and offshore structures, emphasizing the advantages of spectral method over the static method.

**Key words:** Spectral response analysis, displacement, soil reaction, depth factor, free-head pile, fixed-head pile, natural period, natural circular frequency, natural frequency.

## Introduction:

The study of pile foundations under dynamic loading is crucial in geotechnical earthquake engineering, especially in cohesionless soils like sand where soil–structure interaction strongly affects pile performance. The horizontal subgrade modulus ($\eta_h$) plays a key role in defining soil stiffness with depth, influencing displacement, bending moment, and natural frequency. Pile head fixity conditions further alter the response, with free-head piles allowing larger displacements but lower bending moments, while fixed-head piles resist displacement but develop higher bending moment.

During earthquake, piles are subjected to inertial and kinematic forces, leading to complex soil–structure interaction. Dynamic studies show that soil–structure interaction amplifies pile bending during seismic loading, with resonance and head restraint being key factors (Banerjee & Davis, 1978). Fixed heads cause concentrated moments near the top, while free heads lead to larger, deeper displacements (Brown et al., 2001; Rollins et al., 2005). Subgrade stiffness influenced by soil density and saturation reduces deformation in dense sands but increases it in loose or saturated soils (Muthukkumaran & Anbazhaghan, 2006). Continuum and spectral methods give more accurate resonance prediction than Winkler models (Chatterjee & Basak, 2012; Patel & Sahu, 2016). Fixed-head piles increase stiffness but concentrate bending, while free-head piles show larger displacements with smoother moments (Mitra et al., 2023). Spectral studies reveal free heads are more sensitive to stiffness, whereas fixed heads mostly alter bending amplitude, but limited research on their combined seismic effects calls for further spectral investigation (Mukherjee & Banerjee, 2021; Nair & Menon, 2023; Boompandi et al., 2024; Sivapriya & James, 2024).

To address this limitation, the present study investigates the dynamic behaviour of single piles in cohesionless soils using spectral response analysis. The analysis explicitly considers the influence of horizontal subgrade modulus variation with depth along with different pile head fixity conditions (free and fixed). The objective is to quantify how changes in soil stiffness and boundary restraint affect spectral displacement and bending moment response under seismic loading. The outcomes aim to provide improved insights for accurate seismic design of pile foundations, highlighting the advantages of dynamic spectral methods over conventional static approaches.

**2. Methodology:**

In this study, an analytical framework based on spectral response analysis (SRA) was employed to investigate the seismic response of single reinforced concrete piles embedded in cohesionless soils. The piles, with lengths of 12 m, 14 m, 16 m, and 18 m and diameters of 0.3 m, 0.4 m, and 0.5 m, were modelled as micro tower resting on a single pile foundation, where the horizontal subgrade modulus was assumed constant with depth (1.0, 1.5, and 2.0 kg/cm³). A vertical load of 108 tonnes was applied at the pile head, and both free-head & fixed-head and boundary conditions were considered to capture variations in the lateral response.

The soil was modelled as homogeneous, cohesionless, and linearly elastic, while the pile was assumed elastic throughout loading. A 5% damping ratio and only the first vibration mode were considered. The natural period was computed, and spectral displacement from IS 1893 (Part 1): 2016 was used to evaluate pile head deflection, bending moment distribution, and soil reaction. Material properties were adopted from IS 456:2000, pile design from IS 2911 (Part 1/Sec 2):2010, seismic loading from IS 1893 (Part 1):2016, and soil parameters from IS 5249:1992. The semi-analytical model evaluates the effects of pile geometry, subgrade stiffness, and head conditions on seismic performance in sandy soils.

**2.1 Geometric and Material Properties:**

| Parameter | Value |
|---|---|
| Pile lengths (L) | 12 m, 14 m, 16 m, 18 m |
| Pile diameters (D) | 30 cm, 40 cm, 50 cm |
| Modulus of Elasticity ($E_p$) | $2 \times 10^6$ kg/cm² |
| Soil type | Cohesionless (sand) |
| Subgrade modulus ($\eta_h$) | 1.0, 1.5, 2.0 kg/cm³ (constant with depth) |
| Vertical load on pile | 108 tonnes |

**3. Results and Discussion:**

**Fig.3.1** shows the variation of spectral displacement with varied pile lengths. The reduction in spectral displacement is marginal with the length of pile. Piles with 0.3 m diameter showed the largest displacement (~19.7 mm), whereas 0.5 m diameter piles have the lower displacement (~13.3 mm).

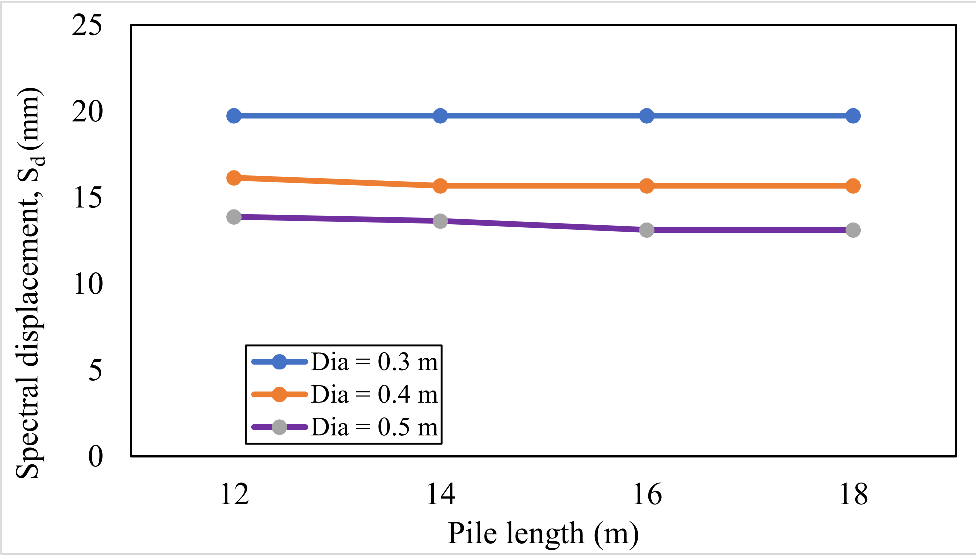


**Fig.3.1 Variation of spectral displacement with pile length at $\eta_h$ = 2 kg/cm$^3$ for free-head pile of different diameters**

**Fig.3.2** shows the maximum variation of bending moment with varied length of pile. As the pile length increases, the bending moment decreases slightly, but higher moment values are observed for larger diameters (0.5 m ≈ 1500 kN-m) of pile compared to smaller diameter ones (0.3 m ≈ 700 kN-m).

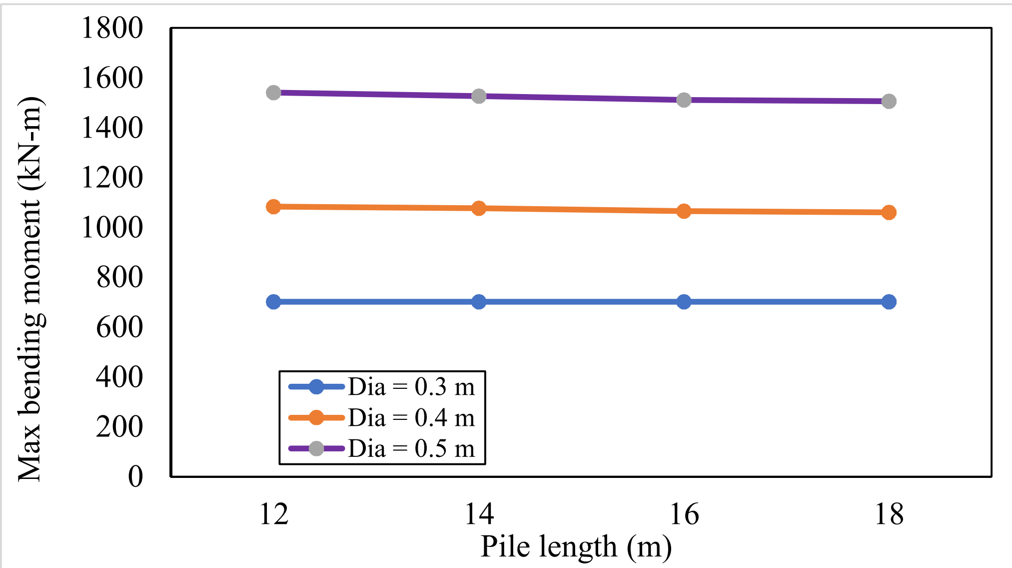


**Fig.3.2 Variation of Max bending moment with pile length at $\eta_h$= 2 kg/cm$^3$ for fixed-head pile of different diameters**

**Fig.3.3** shows the soil reaction with depth for a free-head pile of 18 m length at $\eta_h$ = 2.0 kg/cm³, showing the occurrence of maximum soil reaction near the pile head and it decreased gradually with depth. Among the three pile diameters compared, the 0.5 m diameter pile consistently mobilizes the highest soil resistance, reaching values close to 220 kN/m, while the 0.4 m and 0.3 m diameter piles show progressively lower reactions. These results indicate that the pile diameter has a significant effect on the magnitude of soil reaction, whereas increasing pile length mainly influences the depth distribution. Thus, in free-head conditions at high subgrade modulus, larger diameter piles provide superior lateral resistance.

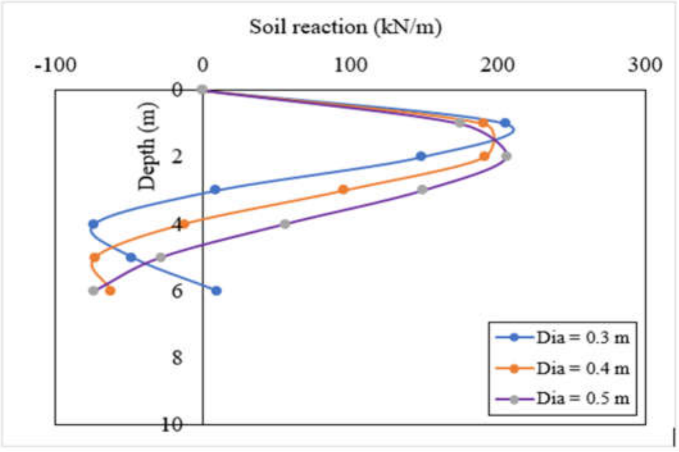


**Fig.3.3 Variation of soil reaction with depth for free-head piles of 18 m length at $\eta_h$= 2.0 kg/cm$^3$**

**Fig.3.4** shows the soil reaction with depth for a fixed-head pile with length 18 m at $\eta_h$ = 2.0 kg/cm$^3$, where the 0.5 m diameter pile shows the highest soil resistance is about 170–180 kN/m near the pile head, while the 0.4 m and 0.3 m diameter piles yielded intermediate and lowest values, respectively. The peak soil reaction is concentrated at the pile top due to the fixed-head restraint and it decreasing quickly with depth. This result highlights the influence of pile diameter on the magnitude of soil reaction, while increasing pile length mainly affects the depth at which resistance is developed. Thus, at high subgrade modulus under fixed-head conditions, larger diameter piles offer superior resistance close to the ground surface.

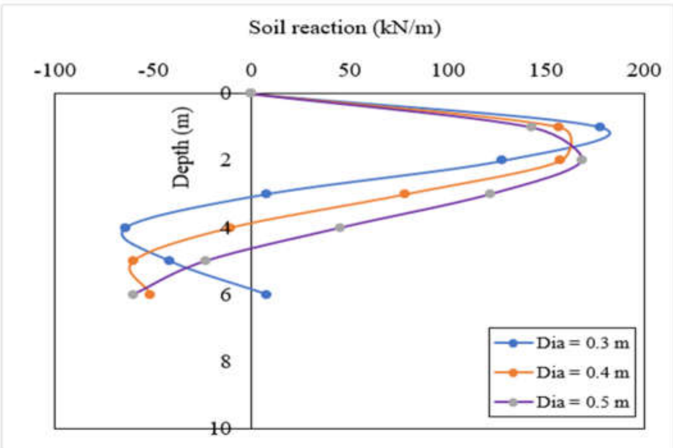


**Fig.3.4 Variation of soil reaction with depth for fixed-head piles of 18 m length at $\eta_h$= 2.0 kg/cm$^3$**

**Conclusion:**

From the spectral analysis carried out on the free-head and fixed-head pile for varied lengths, diameters and modulus of subgrade reaction, the conclusions of first natural period, spectral displacement, maximum bending moment and first natural circular frequency are presented below.

- Increased pile diameter resulted in increased bending moments.

- Variation in pile length resulted a marginal change in the displacement and bending moment for the pile lengths above 14 m.
- A higher horizontal subgrade modulus decreased the displacements but increased the bending moment for fixed-head pile.
- Free-head pile had undergone a larger displacement but experienced smaller bending moments, while fixed-head piles controlled the displacement effectively but developed greater bending moment near the pile head.
- The natural period decreased with increased pile diameter, indicating rigid behaviour, whereas long pile showed slightly higher period of motion.
- Fixed-head pile exhibited shorter natural periods and higher natural frequencies compared to a free-head pile due to rotational restraint.
- Spectral displacement decreased with increased pile diameter, and fixed-head pile consistently showed lower displacements than free-head pile.
- Soil reaction is maximum near the pile head and gradually decreased with depth, with larger diameter piles mobilizing greater soil resistance than smaller ones.
- Increasing pile length does not change the peak soil reaction significantly but alters the distribution of resistance with depth.
- For seismic design, pile diameter is the dominant parameter affecting displacement and soil resistance, while pile length mainly governs depth distribution.
- Designers must balance displacement control keeping in view development of bending moment on the pile.
- Spectral response analysis provides a more realistic evaluation of seismic pile performance compared to conventional static approaches, making it a reliable method for earthquake-resistant foundation design.

**References:**

Banerjee, P. K. & Davis, T.G. (1978): The behaviour of axially and laterally loaded single piles embedded in nonhomogeneous soils. Geotechnique, 28(3), 309–326.

Bharat Bhushan Prasad (2011): Advanced soil dynamics and Earthquake Engineering, ISBN-978-81-203-4039-8.

Boompandi, K., Venkatesh, K., & Muthukumar, M. (2024): Behaviour of piles in sloping cohesionless ground under lateral loads. Innovative Infrastructure Solutions, 9(1), 45–59.

Bowles, J. E. (2000). Foundation Analysis and Design, 5th Edition, ISBN-0-07-118844-4.

Brown, D.A., Morrison, C., and Reese, L.C. (1988): Lateral load behaviour of pile group in sand. Journal of Geotechnical Engineering, 114(11), 1261–1276.

Chatterjee, S. & Basak, P. (2012): Frequency-dependent dynamic response of piles in sand. Soil Dynamics and Earthquake Engineering, 39, 13–23.

Mitra, P., Saha, R., & Basu, A. (2023): Dynamic response of piles in clayey soils under combined loads. Geotechnical Research, 10(2), 105–119.

Mukherjee, A. & Banerjee, S. (2021): Modal analysis of piles with varying subgrade stiffness. Soil Dynamics and Earthquake Engineering, 142, 106–197.

Muthukkumaran, K. & Anbazhaghan, P. (2006): Model tests on laterally loaded piles in dry sand. Geotechnical Testing Journal, 29(6), 1–8.

Nair, P. & Menon, A. (2023): Dynamic lateral response of single piles in layered cohesionless soils. Soil Dynamics and Earthquake Engineering, 169, 107–231.

Patel, R. & Sahu, R. (2016): Spectral finite element analysis of single piles in sand. Structural Engineering and Mechanics, 60(1), 103–118.

Rollins, K.M., et al. (2005): Lateral resistance of full-scale pile group in sand. Journal of Geotechnical and Geoenvironmental Engineering, 131(1), 103–114.

Sivapriya, R. & James, N. (2024): Laboratory study on pile groups near slopes under lateral load. International Journal of Geomechanics, 24(3), 04024021.

IS 5249:1992: Determination of Dynamic Properties of Soil- Method of Test.

IS 1893: Part 1: 2016: Criteria for Earthquake Resistant Design of Structures – Part 1: General Provisions and Buildings.

IS 456:2000 Plain and Reinforced Concrete Code of Practice.

IS 2911-1-2 (2010): Design and Construction of Pile Foundations - Code of Practice Part I: Concrete Piles, Section 2, Bored Cast In-situ Concrete Piles.